6 Economic Theory and Mathematical Method: An Interview

Feiwel Would you say that Arrow has a mathematical bent of mind?

Aumann Absolutely. He is an extraordinarily clear thinker. His mathematical acumen never ceases to amaze me. Very deep and complex kinds of mathematics are not his forte, but he both understands and is able to produce subtle mathematical arguments; he is a very good mathematician. Several of the most important ideas of this century in mathematical economics are due to him.

Take the existence of competitive equilibrium, which has by now become a commonplace. When it was discovered in the early 1950s it was an extraordinary tour de force of deep mathematical thinking. Of course, that was done by Arrow and Debreu, but it was actually discovered independently by each. That idea alone is one that identifies Arrow as a first class mathematician—the very idea of applying a fixed-point theorem in the way in which it is applied, the making of that connection, that is an extraordinary idea.

In my opinion the important mathematics is not necessarily the most complex, involved gymnastics that one can do. The really important pieces of mathematics are those that can be reduced to at most a few pages. An idea that is more complicated than that will eventually be forgotten. Arrow's mathematical work falls into that lasting category of things that are very far from obvious but are still basically simple—simple, not in the sense of easy, but in the sense of clean, like the best modern architecture.

Feiwel How have these aesthetic and profound mathematical ideas improved our comprehension of the economy and society?

Aumann Your question, of course, does not relate to Arrow in particular but to all of mathematical economics. It is a subject on which one cannot say much that is sensible in the framework of an interview. In *Frontiers of Economics* (edited by Arrow and Honkapohja) I have an article entitled "What is Game Theory Trying to Accomplish?" in which mathematical economics is also treated. Very briefly, if one tries to do economic theory without using mathematics—that is, purely verbally—one gets the impression that one can reach any conclusion. Anything can be done with words, but afterwards one is not sure where one stands, what has been shown, what has not been shown, and what it is all about. When one presents economic theory in a formal model, then one can show that certain assumptions necessarily lead to certain conclusions. Mathematics imposes a discipline of thought; it forces one to think clearly.

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Mathematics has been called the language of science. That also sums up its relation to economics very well.

There is another aspect that bears mention, and that is the interdependence, what could be called the ecology. In an economy, like in a biological environment, all things hang together; change one, and everything else will change, usually in ways that are far from obvious. One of the tasks of mathematics is to try to see, at least qualitatively, where such changes might go.

Of course, one must add immediately that once one has a model, and one has a set of assumptions and a conclusion, if it remains in mathematical form, it is not worth very much. You have to be able to translate it back into words. But then at least you have something to back up the words, so that if someone asks you, "Now just what do you mean by that?" it is easy to say precisely what you do mean. If you cannot explain it in words, it is not worth very much; but if you can *only* explain it in words, it also is not worth very much.

Feiwel Is there a fundamental difference in approach between game theory and g.e.t.?

Aumann There are probably two fundamental methodological differences. The more important one is that the theory of games is much more general. It refers to any situation of human interaction. A specific situation like a market would be an application of the general theory of games. G.e.t. refers only to a specifically defined situation and does not apply any further than that. So that the domain of game theory is much more general. The other, less important, difference is that by definition, g.e.t. is not specifically interactive; that is, the protagonists react to prices rather than reacting to each other in the way that they do in game theory. What differentiates game theory from economic theory in general is that economic theory consists of a number of specific models tailored to specific situations with no wider applications, whereas game theory is a sort of unifying theory that in principle covers everything.

Feiwel What do you think has been the influence of game theory on Arrow?

Aumann Arrow has done a lot of work in applications of non-cooperative game theory to specific economic environments, as distinguished from works in a general game theoretic set-up. The only one of that kind—and it is, of course, an extremely significant exception—is the work on social choice theory, which is basically axiomatic, cooperative game theory. It is, of course, one of the great milestones of his life.

Arrow is fundamentally an economist. He is interested in economic problems. He has contributed an enormous amount to information eco-

nomics, and that is very closely tied into game theoretic ideas. So there is another link.

Feiwel Are the young mathematically sophisticated economists shifting away from g.e.t. towards game theory?

Aumann There is some of that. Arrow would be one of the first to emphasize the limitations of the g.e. model. In fact, he feels a little uncomfortable with the g.e. model, also because of its anti-socialist implications. The idea that the market leads to efficiency is something that is foreign to Arrow's emotions, so that he is happy to see the limitations of the model and to see that there are many ways in which the market can fail (information, public goods, what have you). He emphasizes that one of the lessons of modern economics is the manifold ways in which the market mechanism can fail to yield efficiency, the ways in which the imperfections manifest themselves.

As I said before, game theory provides a tool that allows one to analyze just about any interactive situation; that would include all the various aspects of market imperfections that will lead away from efficiency. All these are amenable to game theoretic analysis. Perhaps this is a reason for the shift. Basically, game theory is a wider ranging tool than the g.e. model, which is quite limited in the kind of situations with which it can deal.

Feiwel Would you care to comment on Arrow's views of the selfish motive, cooperation and trust?

Aumann There is one theme in game theory that lately has been getting increased attention and that is: What is the connection between selfishness and cooperation? Can one derive cooperation from purely selfish motives? That is related to the interaction that has recently been growing between evolutionary biology and game theory. One would like to think of the selfish motive and cooperation not as being opposed to each other, but as dovetailed; of cooperation as the consequence of selfishness.

But it is not at all certain that Arrow would go along with that. His point of view, as I read it, is that cooperation, helping other people, the socialist ideal, that sort of thing, is a supreme value that he would like to take as a starting point. Whereas he is too wise to take it for granted that everybody behaves in this way, there is something deep inside him that hopes that people do behave that way, and he sometimes acts as if that hope were a reality.

Here is an anecdote: An issue that comes up perennially in the Econometric Society is the method of electing fellows. That, of course, is a jewel of an issue for the Econometric Society; it is social choice in the home of the social choice experts. Lately one of the issues that has come up in that

connection is that of strategic voting. This means taking into account how others might vote, rather than simply voting one's true preference. For example, in the US in 1980, somebody who preferred Anderson, but voted for Carter because he thought Anderson had no chance, was voting strategically.

There is a sort of counterpart to Arrow's famous impossibility theorem, due to Gibbard and Satterthwaite, that says that whenever there are more than two alternatives, *any* non-dictatorial election method is subject to strategic voting. In addition to its great theoretical interest, this has important practical implications. Strategic voting introduces noise into the system; rather than voting what they think, people try to outguess the other voters. The result may well involve significant distortions. Practically, one would like to build the incentives so as to minimize these distortions.

Specific, practical, real-life implications of theory have always fascinated me; the problem of minimizing strategic voting in the election of fellows caught my imagination. It seemed paradoxical and incongruous that just the Econometric Society would ignore incentive effects in its own voting mechanisms; the message seemed to be, "theory is OK for making a living, but for heaven's sake, let's not take it seriously in practical affairs." I discussed the matter with Arrow, expecting to find a sympathetic ear from the founder of social choice theory. He did express interest, but little real enthusiasm for the issue. He himself does not vote strategically; somehow he feels that it is unbecoming that, as scholars and gentlefolk, the Fellows of the Society should engage in such practices. I have assured him that I do; I play the game by its rules and I see nothing even remotely immoral or unethical about it. But whereas intellectually he recognizes that some people do vote strategically, emotionally there is something in him that rejects this. He does not want to bother with that kind of thing and expressed surprise at my involvement. As an economist he recognizes the importance of incentives; but as a humanist, he cannot get terribly excited about them in a practical context.

I have dwelt on this at some length because it illustrates nicely the dualism, the intellectual tension in the man.

Feiwel Does game theory assume much greater rationality of agents than g.e.t. and if so why?

Aumann No, I would not buy that. I think that the amount of rationality assumed is basically the same. Obviously it is too much. One of the purposes of the 1985–86 programme at the Mathematical Sciences Research Institute (Berkeley), of which Arrow is a member, is to explore the consequences of bounded rationality. But before one does that, one

has to formulate precisely the idea of bounded rationality. On the other hand, perhaps full rationality is not such a bad assumption; it is a sort of idealization, like the ideas of perfect gas or frictionless motion. Idealizations have always been very fruitful in science. To some extent people do behave rationally; the idea of full rationality is no less valid than any other scientific idealization.

Feiwel Specifically, what do game theorists mean by rational behaviour?

Aumann Utility maximizing behaviour, just as economists do. Each person maximizes against the given situation as he perceives it. This does not imply full information; no matter what your information is, you have to have some estimate as to what might happen, and you maximize against that.

Feiwel Am I correct that you are not overly impressed with 'satisficing' and 'bounded rationality' in Herb Simon's sense?

Aumann My criterion for judging any piece of science is how effective it is, where it leads, whether it leads to insights, to a considerable body of work, to better understanding. There are two different criteria: One is, how plausible are the assumptions? The other is, where do the conclusions lead? Many people use the first criterion—*a priori* plausibility of the model—as the criterion for judging a piece of work. But I prefer the second criterion: Where does it lead? The usual assumptions of utility maximization have led to practically all of economic theory, and at least some of that helps us to understand the world. The idea of satisficing and other ideas of that kind that abandon the model of full rationality are extremely attractive as hypotheses. If I ask myself, do I satisfice or do I maximize utility? I have to answer that I satisfice. So the concept is very attractive as an assumption, but I do not know of any general conclusions to which it leads. It is not that the concept *cannot* lead to anything. A coherent model of bounded rationality could very well lead to interesting results, and people are beginning to generate such results now. But up until now they had not led to any significant body of theory or indeed to anything very startling.

Feiwel Did game theory revolutionize our understanding of how the economy really works?

Aumann The revolution has been in method, in the way we think about economic problems. Game theory is a tool, not a product. There have been hundreds of applications. Take the idea of Nash equilibrium; it is a method, a tool of analysis; it is not in itself an economic insight, but it leads to economic insights. For example, there is an enormous amount of interest in auctions nowadays. How do you analyse auctions? You formulate them as incomplete information games and you look for their

equilibria; Nash equilibria, perfect equilibria, sequential equilibria, and various related ideas. You apply them to specific, real-life auctions and you get outcomes. These things are very important. Oil-lease auctions are held in which the values of the properties easily reach 100 million dollars in one auction. Game theory contributes in a very practical, down-to-earth way, in addition to providing general, theoretical insights.

We talked before about Arrow's work in game theory. One of the pieces of work he did about a dozen years ago was the application of game theory to analyzing racial discrimination in the job market. It is enormously important to understand what part of discrimination is just cussedness and ignorance, and what part of it is really rational given the circumstances. And if it is rational given the circumstances, how can we change those circumstances? What can we do about it? How can we change the situation, so that the incentives are structured against discrimination? The tools one uses are game theoretic.

Game theory also enables one to attack better such problems as the economics of health insurance, labour relations, etc. In cooperative game theory you have core analysis. For example, Al Roth recently discovered that the way that interns are assigned to hospitals in the US is described exactly by an algorithm for finding a point in the core of the corresponding market. Another example is the application of the Shapley value to voting situations where you try to get representation for various districts in accordance with the "one man—one vote" rule. It is a methodology that enables you to analyze all kinds of situations, and in that sense it has indeed revolutionized our ways of thinking.

Feiwel Since your work on the continuum of traders figures prominently in this volume, can we overcome your natural reluctance to talk about yourself and explore the creative genesis of this work?

Aumann One day I received in the mail an article written by Milnor and Shapley—an analysis of voting in a situation in which there are some large voters and what they called an "ocean" of small voters. Afterwards Shapley wrote an article applying this to a corporation with two large stockholders; the many small stockholders constitute the "ocean." It was an analysis, using the Shapley value, to investigate power relationships. The idea caught my imagination; it was a beautiful paper. This was in the winter of 1960–61. Then in the fall of 1961 there was a conference on Recent Advances in Game Theory at Princeton University. Herb Scarf gave a paper there that was a forerunner of the Debreu-Scarf paper on the core of an economy, and an outgrowth of previous work by Shubik (and by Edgeworth). Scarf's model had a denumerable infinity of traders, divided into a finite number of types, and he got an equivalence theorem between the core and the competitive equilibrium. However, this

model had various defects. For example one had to be careful about how one defined the sum for this denumerable infinity. I remembered the paper by Milnor and Shapley about "oceanic" games when hearing Scarf's model and said to myself, "surely, the continuum just *has* to be the right way of doing that." It was really putting these two ideas together that was the genesis.

A few minutes ago we were discussing applications of game theory. The continuum was a purely game-theoretic idea. The way Milnor and Shapley originally thought of it was not at all in an economic context and had nothing to do with the core. Then came Scarf's model of the core in an economic context. So ideas coming from one place fit nicely into another, completely different, place. That kind of interaction is one of the most important ways in which game theory contributes to economics. Since game theory is not constructed with a specific application in view, it is sufficiently broad so that ideas from one context apply to others as well. One is able to tie things together, to see the common underlying principles. Indeed, it is one of the significant ways in which science operates in general—making connections, seeing the big picture.

Feiwel In your paper "What is Game Theory Trying to Accomplish?" you have a wonderful passage about game theory and mathematical economics as art forms. Could you enlarge on this idea?

Aumann The best art is something that strikes a chord with the viewer or listener. It expresses something that the viewer or listener has experienced himself and it expresses it in a way that enables him to focus his feelings or ideas about it. You read a novel and it expresses some kind of idea with which you can empathize, or perhaps something that you yourself have thought about or experienced. Take a sculpture or a cubist painting. It expresses some reality, some insight, in an ideal way. That is what the best mathematical economics does. It is a way of expressing ideas, perhaps in an ideal way.

Feiwel In the same paper you mention that "our fields are by no means the only ones in science that are not strong on predictions and falsifiability; in which the measure of success is 'does it enable me to gain insight?' rather than 'what will be my observations?' Similar in this respect are disciplines like psychoanalysis, archeology, evolution, meteorology and to some extent even aerodynamics." Can we explore this further?

Aumann This is a theme that Arrow has expressed repeatedly. One of his favourite examples is meteorology. We are good at explaining what makes the weather, but we are not very good at predicting it. We have not become much better since Arrow did meteorology in the Second

World War. We are significantly better, but the significance is only three or four percentage points. With all our satellites and the like we still cannot predict the weather. And that is what is happening in economics. Perhaps we understand economics a little better, but we are still not very good at predicting what will happen.

In this connection, let me tell you an anecdote. I was on the Hebrew University committee that oversees the doctoral theses in the experimental sciences. I came across this thesis in meteorology explaining and predicting the weather on a perfectly round island in an otherwise empty ocean extending over an infinite plane. Well, I thought, that is wonderful. Like many people in economic theory, I have a tendency to breast-beating. After seeing that thesis, I thought, "welcome to the club." The author was not at all bothered by such questions as the realism of assumptions. This kind of work makes an important contribution, however, because once we understand how the weather behaves in such a situation, perhaps we can understand how the weather behaves in different situations, where the island is not perfectly round and the ocean not infinite. Perhaps it provides us with some qualitative insights that are applicable elsewhere.

Feiwel A few sentences before the cited paragraph you mention that "the sciences are the children of our minds; we must allow each one of them to develop naturally, and not force them into molds that are not appropriate for them." Could you enlarge on that theme?

Aumann We should not try to think of economics as physics or chemistry. Ernst Mayr has written a book entitled *The Growth of Biological Thought* (which, by the way, I first saw in Arrow's office and borrowed from him). Mayr makes the point repeatedly that biology is not physics and that one should not try to apply to it criteria like falsifiability, that grew out of the philosophy of physics. They are inappropriate for biology, and I think that they are inappropriate for economics also. One cannot get very far applying that sort of criterion in economics. We have to understand that in physics one generally expects a unique result from any given situation. Even in theory one cannot make that kind of prediction in economics. Thus we often have situations where all kinds of circumstances, in addition to those described as economic ones, are operative and are going to affect the outcome. When you have a competitive equilibrium that is not unique, how can you say even in theory which one takes place? A lot of economics involves index numbers of various kinds, indexes in the sense of averages. You can say something about how you estimate a situation without being able to make a clear prediction.

There are many areas where falsifiability is not the criterion. When you study evolutionary biology you are totally unable to falsify anything.

You are explaining the past; it is important to understand the past also. Falsifiability is definitely not the only possible criterion for a useful scientific theory.

Feiwel I understand that in physics the fundamental question of existence of equilibrium was only explored much later after the concept of equilibrium had been in use for some time (similarly as in economics). Can we have your reflections on the importance of the question of existence in economics?

Aumann Existence is an important issue, but not a *primary* one. Your model must have interesting substantive implications before it makes sense to study existence. Sometimes people introduce a new concept, discuss the definition a little, prove existence, and then call it a day. I do not find that kind of work very interesting.

Once one has a model or concept of established interest or usefulness, like competitive equilibrium, then it becomes very interesting and even vital to establish conditions for existence, to delineate the domain of the concept. It is the boundary of the domain that is important. Non-existence is as important as existence; one needs to understand what kind of conditions can be destabilizing. In the study of competitive equilibrium, it is as important to know that non-convexities or discontinuities can lead to non-existence as to have the existence theorem itself.

Feiwel Are there any developments in mathematics at present that are likely to have a profound influence on economics?

Aumann One answer is suggested by the particular marriage we have this year (1985–86) at the Mathematical Sciences Research Institute (Berkeley) between mathematical economics and complexity theory; the implication being that complexity theory—a relatively recent development in mathematics—will have an important influence on economic theory. But I think that it is often very difficult to tell beforehand what kind of discipline will have a significant influence. In the past we have had the influence of global analysis, differential geometry, measure theory, convex analysis, and the like, and nobody could have guessed it before it happened. It is really difficult to foretell.

Feiwel What is complexity theory?

Aumann Basically it is concerned with the difficulty or length of time involved in doing some kind of algorithm. How fast can one solve some kind of problem, such as linear programming, for instance? How many additions and subtractions does it take? Complexity usually has to do with computations of various kinds. In other words, if you are given a problem, what is the maximum number of steps that might conceivably be required to solve that problem? An example of the kinds of ways

in which complexity theory may be useful in economics could be the following: If you limit your agents to being able to perform only a certain number of steps, and they have to reach a decision based on that, how would they do it? What would be the best kind of decision they could reach?

Feiwel Is computer technology involved?

Aumann In theory yes, because when, for example, you are asking about the complexity of linear programming, it gives you an idea of how large a computer you have to have to solve the problem. But sometimes the algorithms suggested by complexity theory are, for one reason or another, not practical. Sometimes they are, but often not. Complexity theory is a sort of theoretical background for computer technology. The relationship is somewhat similar to that between microeconomics and macroeconomics.