Quantum/Classical Correspondence in the Light of Bell's Inequalities

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Instead of the usual asymptotic passage from quantum mechanics to classical mechanics when a parameter tended to infinity, a sharp boundary is obtained for the domain of existence of classical reality. The last is treated as separable empirical reality following d'Espagnat, described by a mathematical superstructure over quantum dynamics for the universal wave function. Being empirical, this reality is constructed in terms of both fundamental notions and characteristics of observers. It is presupposed that considered observers perceive the world as a system of collective degrees of freedom that are inherently dissipative because of interaction with thermal degrees of freedom. Relevant problems of foundation of statistical physics are considered. A feasible example is given of a macroscopic system not admitting such classical reality.

The article contains a concise survey of some relevant domains: quantum and classical Bell-type inequalities; universal wave function; approaches to quantum description of macroscopic world, with emphasis on dissipation; spontaneous reduction models; experimental tests of the universal validity of the quantum theory.

Contents

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I do not believe in micro- and macrolaws,
but only in (structural) laws of general validity.

A. Einstein[3]

Introduction

Sixty years have gone by since N. Bohr read his historic Como lecture[29], where the Copenhagen interpretation of quantum theory was given, and fifty years since A. Einstein, B. Podolksy, and N. Rosen published their basic paper[70] where a fundamental inconsistency of quantum and classical ideas on correlations came to light and doubts were expressed as to the completeness of the quantum theory, whose signal success has since become overshadowed by "accursed questions" on the relation between quantum and classical descriptions. To construct a quantum theory of the macroworld was considered impossible. At the same time, one failed and still fails to find any definite limitations for the applicability of the first principles of quantum theory. Initially stated in the domain of atomic spectroscopy, those principles brilliantly passed the most varied tests made using rapidly progressing technical means[2, 246, 186]. Today the construction of a micro- and macroworld quantum theory is considered an important goal, increasingly attractive to theoretical and experimental physicists. In this context, intensive discussions and re-interpretations of quantum theory's foundations are taking place[1, 186, 156, 92, 157, 120, 200, 63, 59].

Following the publication of [29, 70], the mutual opposition of various classical concepts (Newtonian particle dynamics, Maxwellian field dynamics, Einsteinian geometrodynamics) has moved into the background, while their joint opposition to quantum concepts has been put in the forefront. Corpuscles, waves, and even determinism are only details, the chief thing being a generalizing notion introduced in [70], namely, local elements of a physical reality. Einstein persistently strove for a universal theory which should be completely based on such elements, and considered the quantum theory as a tentative deviation from this concept—pointing out[71] that among the known facts none excluded, in principle, a return to it. Meanwhile, such a fact was predictable, not far from the thought experiment proposed in [70], and was discovered by J. Bell[22] in 1964 as a result of his deep revision of the classical concept which now began speaking not only in words but in formulas.

[3] Quoted in [87, p. 26].

The Copenhagen interpretation requires a combined application of two intrinsically alternative ways—classical and quantum—of describing the physical world. Accordingly, the world is represented as broken in two by a "conceptual gap," so that stages of preparing and registering the quantum objects require classical (rather than quasi-classical) apparatuses. Bell expressed his attitude toward the situation [20, p. 177] by the words "the infamous boundary." Progress in modern technology permits observation of some quantum effects for macro-objects (see reviews [167, 174]) which makes it necessary to discuss fundamental problems of quantum theory from a less academic standpoint. (It is interesting to recall some conference titles[186, 120], such as "The foundations of quantum mechanics in the light of new technology.") The rehabilitation of a unified interpretation of the physical world and the solution of basic problems of quantum-theory foundations would be possible if, acknowledging this theory to be applicable to all objects of the physical world ("principle of presumption of a quantum description"), one could demonstrate that, given suitable conditions and objects, one obtains a classical (rather than quasi-classical) description within the quantum theory (a "reconstruction of the classical world"). In this case Bell's inequalities could be used as a criterion for the applicability of the classical description. The present article is largely concerned with possible approaches toward such a solution of the fundamental problems of quantum theory. Within these approaches there exist real macroscopic quantum effects and we give an explicit indication as to the conditions (low temperatures in particular) where they occur. If verified, such effects will fundamentally change our picture of the macroscopic phenomena that surround us—which have so far been described only classically. It is difficult to overestimate the potential for quite new practical applications—and perhaps for explaining, within quantum theory, the biological phenomena which possess an essential wholeness peculiar also to quantum phenomena.

The point of this article is, above all, to join the following three ideas: (1) quantum theory is universally applicable, and classical reality may be reconstructed (reconstituted) from quantum dynamics; (2) the characteristic of classical reality is independent existence of objects, absence of quantum correlations between them, and fulfillment of classical Bell-type inequalities; (3) dissipation is the fundamental relationship between collective degrees of freedom, admitting the classical description, and thermal degrees of freedom constituting the environment.

The first result of this conjunction was unexpected. Instead of the usual asymptotic passage from quantum mechanics to classical mechanics when a parameter tended to infinity, a sharp transition was obtained at some value of the parameter. It seems that until now such a situation has occurred only in phase transition theory. The exact wording is given in Section 4. Here we outline only one of our final formulas with minimal explanation: the classical description is applicable when

$$\frac{A}{\Gamma T} \leq 2\frac{k_B}{h}$$

here A is the coefficient of elasticity, measuring a potential interaction of two bodies: $U(q_1, q_2) = \frac{1}{2}A(q_1 - q_2)^2$; Γ is the drag coefficient measuring a dissipative interaction of the body with its environment: $F_{\text{fric}} = -\Gamma\dot{q}$; T is the temperature of the environment; k_B is Boltzman's constant, and h is Planck's constant. Note the sign $\leq$ instead of the usual sign $\ll$.

Each of the three ideas above has its history, briefly dealt with in Sections 1–3. Section 1 surveys investigations of correlation functions related to Bell-type inequalities. A part of Section 3 is a survey of relevant investigations of dissipation. Special attention is paid to the idea of reconstructing classical reality (Sections 2 and 3). The fact is that for the last decade, discussions of the quantum/classical correspondence assumed a somewhat evasive character: authors keep developing formalisms, but evade explicitly accepting a certain interpretation. This is seemingly a response to developments in former decades, when the interpretation problem was repeatedly proclaimed solved, but nevertheless is still considered unsolved (see Section 3). Concerning the origin of this situation, we agree with D'Espagnat[58] that it is the indistinct status of the notion of "classical reality": whether it is treated as absolute or relative (independent or empirical, in terms of [58]). In other words, whether "real" means "real in itself" or "real relative to a class of observers". In order to reconstruct absolute (independent) reality, defining this notion in terms of fundamental notions only is necessary. However, this scheme seems to fail. A notion of relative (empirical) reality needs to be defined using both fundamental notions and characteristics of observers. We meet the challenge of D'Espagnat and think that the title of his article could fit ours as well (his question mark being replaced by an exclamation mark): Toward a separable empirical reality!

One can easily get a contradiction if one confuses concepts belonging to essentially dissimilar approaches. For instance, one can reject our reasoning, claiming that a quantum system does not became classical just because it satisfies Bell's inequality in some specific situation. To avoid such misunderstandings, we consider it necessary to give an introduction to the "relative reality approach." Starting with the most general subjects concerning the universal wave function (Section 2), we proceed gradually to concrete tasks (Section 3). Thereby we hope to convince the reader that our contribution to posing and solving these concrete tasks may be considered a contribution to general problems, as well. We hope also that the reader will not be alienated by the fact that our sections are quite different in style. Thus, in Sections 1, 4, and 6 the emphasis is on rigorous statements, using well-known formalisms. Sections "without formulas" (i.e., Sections 2, 5 and partly 3) are concerned with discussions of first principles. Section 4 mainly contains our new results. Other sections discuss works published earlier, from the classical to the contemporary.

Unfortunately, the wide scope, resulting from the combining nature of this work, forces the review sections to be concise. If the reader is not acquainted with the areas discussed, he will hardly became so by reading our review sections. They are intended for a reader familiar with some of the areas and interested in others. Such a reader should be able to use our review as a guide to the literature. To understanding the meaning of our new results (presented in Section 4) one needs only to come to grips with the essential ideas of each section, rather than studying each in detail. The details would hopefully be useful for further work in this area (which is as yet far from complete).

Some recent attempts to restore absolute (independent) reality by replacing unitary quantum dynamics with some hybrid quantum/classical ones are considered briefly in Section 5. In contrast, we are not intending to touch up unitary quantum dynamics. Accordingly, to avoid a circular proof, statistical physics used for describing dissipation needs a foundation without approximations which misrepresent quantum dynamics by transforming pure states into mixed ones. An approach free from such approximations is considered in Section 6. The last section, Section 7, is concerned both with experiments on verifying the classical Bell inequalities or their quantum analogs and with possible experiments on macroscopic quantum effects. Under very careful, but undoubtedly feasible isolation of the collective degrees of freedom from the thermal ones, quantum correlations can arise and be conserved for long periods of time, even in the mechanical motion of macroscopic bodies. The required degree of isolation may be calculated by the criterion given in Section 4.

1 Kinematics and Dynamics of Correlations

Before 1964, the investigations initiated by Einstein, Podolsky, and Rosen [70] were pursued only by "atypical theoreticians," or those in "an atypical state" outside the quantum paradigm (see the review in [21]). After Bell's pioneering work[22], the problem was dealt with by some experimentalists (see the reviews [112, 7], which contain also an elementary introduction). However, "typical theoreticians" took Bell's work as a "calculus of chimeras," as in a subjunctive mood: *if* there existed hidden variables, the integration over them would result in some (incorrect!) inequalities. Does it pay to master the art of estimating integrals over hidden variables which do not and cannot exist? In 1978 one of us (Tsirelson) reported on Bell's inequalities to a seminar whose chairman, A. M. Vershik, asked him the question, "Are similarly general inequalities possible in the quantum theory?" This question was answered in [51], which contains quantum analogs of Bell's inequalities (later rediscovered[227]) obtained by an operator technique that is far from a "chimera." The classical Bell inequalities can be considered as those for commuting operators[51]. Another situation to which they apply within the quantum theory was known; namely, they are fulfilled for factorized states and their mixtures (the first indication of this fact known to us can be found in [216]; see also[228, 11]). Thus, after Bell's inequalities had played a historic role in rejecting local hidden variables, they found a second life within the quantum theory as one of the "crystallization centers" of a quantum correlation theory (see [51, 227] and [228, 11, 235, 147, 160, 161, 162, 229, 230, 146, 158, 159, 93, 241]).

In this scantily explored field some selected problems are under study, while others await investigation. There are problems within the framework of general principles of quantum theory, that of the axiomatic local quantum field theory, and that of the free-field theory. There are, of course, other variants. Note especially two, namely one lying beyond the framework of the quantum theory, and one embedded in the classical theory; these will be dealt with below. Another important classification feature is presented by single-time and multiple-time problems. We relegate them to the kinematics and dynamics of correlations, respectively. The content of these particular branches of science, as we see it to date, is presented in this section. We will start with kinematics of correlations within the framework of the principles of general quantum theory.

Let commuting subalgebras $\mathcal{A}_i$ be singled out inside the algebra of quantum observables, and consider some observables A_{ij} from each $\mathcal{A}_i$. So, $A_{i_1 j_1}$ and $A_{i_2 j_2}$ must commute when $i_1 \neq i_2$ but must not if $i_1 = i_2$. Suppose for simplicity that each A_{ij} has a discrete spectrum. Denote its eigenvalues as a_{ijk} and spectral projections as P_{ijk}. Fixing some quantum state (here and below it is not explicitly indicated in many cases, but only implied in the notation $\langle \dots \rangle$ for mean value of an observable), one will obtain a probability distribution

$$P_{j_1 j_2 \dots}^{k_1 k_2 \dots} = \langle P_{1 j_1 k_1} P_{2 j_2 k_2} \dots \rangle \tag{1.1}$$

This many-indexed value is the probability of a coincidence of the following events: the measurement of the observable A_{1j_1} has given the result k_1, that of the observable A_{2j_2} the result k_2, etc. The problem is to determine all those general conditions which are fulfilled for any system which can be described in this way. This, of course, is an enormous task. In practice, much more restricted problems are studied.

Let each index i, j, k take on only two values. This is the simplest nontrivial case. It is convenient to assume that $a_{ijk} = \pm 1$, i.e., $A_{ij}^2 = 1$. The set of 2^4 (=16) numbers obtained from (1.1) is easily reduced to one of eight numbers involving four mean values $\langle A_{ij} \rangle$ and four correlations $\langle A_{1j_1} A_{2j_2} \rangle$. Finding a complete system of inequalities for those numbers is equivalent to describing the proper geometrical figure in eight-dimensional space. Nobody has succeeded in obtaining its explicit equations. There is an implicit description (Theorems 2 and 3 in [51]) which is cumbersome, but at least is expressed by a finite number of scalar equations rather than operators of an indefinite kind in a Hilbert space. The inequalities for the general case turn out[51] to be indistinguishable from the case of correlated spin-$\frac{1}{2}$ particles, which is well known to both theoreticians and experimentalists. To simplify the problem further, we shall look for the restrictions on correlations while regarding mean values as arbitrary (or assume mean values equal to zero; these two cases are equivalent[51]). In other words, we are required to find a four-dimensional figure which is a projection (or, which is the same in this case, a section) of the above-mentioned eight-dimensional figure. An explicit solution of this problem, expressed in sixth-degree polynomials, was suggested by one of the authors[235]. Another explicit solution independently found by Lawrence Landau[158] takes a surprisingly simple form when the correlations are written as cosines of certain angles. If we take into account the geometrical interpretation of such angles indicated in [51, point 4 of Theorem 1], the question will reduce to a theorem of spherical geometry on the existence of a spherical quadrangle with sides of a given length.

The combination of four correlations considered in [48],

$$R = \langle A_{11} A_{21} \rangle + \langle A_{11} A_{22} \rangle + \langle A_{12} A_{21} \rangle - \langle A_{12} A_{22} \rangle \tag{1.2}$$

merits attention for reasons which will be elucidated below. There is a known inequality [51],

$$|R| \leq 2\sqrt{2} \tag{1.3}$$

which may be obtained from the general relations mentioned above. There are also several direct proofs which we shall list now. Suffice it to show that the spectrum of the operator $C = A_{11}A_{21} + A_{11}A_{22} + A_{12}A_{21} - A_{12}A_{22}$

lies on the segment $[-2\sqrt{2}, 2\sqrt{2}]$. The first proof (close to that given in [51]) follows from the equality

$$\begin{aligned} 2\sqrt{2} - C &= \frac{1}{\sqrt{2}}(A_{11}^2 + A_{12}^2 + A_{21}^2 + A_{22}^2) - C \\ &= \frac{1}{\sqrt{2}}\left(A_{11} - \frac{A_{21} + A_{22}}{\sqrt{2}}\right)^2 + \frac{1}{\sqrt{2}}\left(A_{12} - \frac{A_{21} - A_{22}}{\sqrt{2}}\right)^2 \end{aligned} \quad (1.4)$$

The second proof[147, 160] is based on the equality

$$C^2 = 4 - [A_{11}, A_{12}] \cdot [A_{21}, A_{22}] \quad (1.5)$$

and the trivial bound$\|A_{i1}A_{i2} - A_{i2}A_{i1}\| \leq 2$. The third proof, recently found by Landau,[4] is based upon the equality $C^2 + D^2 = 8$, where

$$D = A_{11}A_{21} + A_{11}A_{22} - A_{12}A_{21} + A_{12}A_{22}. \quad (1.6)$$

It is well known[48] that the equality $R = 2\sqrt{2}$ is attained in a spin-correlation experiment following Bohm's scheme [25, point 22.16]. Are there other cases of attaining the equality in relation (1.3)? Essentially, no. The situations unitarily equivalent to the situation above and their direct sums (with the possible addition of unimportant direct summands) exhaust all cases of obtaining the equality[235].

Returning to more general cases, let i and k assume two values as before, but let j run through any finite set of values (possibly depending on i). There are results on the correlations $\langle A_{1j_1} A_{2j_2} \rangle$ with unspecified or, which is equivalent, zero mean values. The corresponding class of correlation matrices was described[51, 235] in terms of scalar products of unit vectors in Euclidean space (instead of operators in the Hilbert one). This class of matrices will not be affected if the observables A_{ij} are subjected to canonical anticommutation relations, i.e., if we require that all symmetrized products $A_{ij_1} \circ A_{ij_2}$ be scalar multiples of the unit operator. Thus, a proper generalization of Pauli spin matrices is given here by the Clifford algebras rather than the higher spins, as one might have thought. The result mentioned above, which defines the spin observables as the only (in principle) solution of the extremal quantum correlation problem, is also generalized to the Clifford algebras[235], the term "extremal" being understood in a more general sense.

For the case where i takes on three values, and j, k two values each,

[4] We are obliged to L. Landau for communicating this result to us.

there are only a few special results [235, 189, 231]. D. M. Palatnik[189] has considered the following analog of (1.2) for triple correlations[5]:

$$R_3 = \langle A_{11}A_{21}A_{32}\rangle + \langle A_{11}A_{22}A_{31}\rangle + \langle A_{12}A_{21}A_{31}\rangle - \langle A_{12}A_{22}A_{32}\rangle \quad (1.7)$$

having found that a proper choice of observables and state may give the equality $R_3 = 4$ [for pair correlations, this value is forbidden by inequality (1.3)]. This fact opens the way for "Bell's theorem without inequalities"[93]. One of the several results obtained in [235, Sect. 5] is

$$R^2 \leq 8 - 2(\langle A_{2j_2}A_{3j_3}\rangle)^2 \quad (1.8)$$

for any j_2, j_3. This shows that an extremal correlation between two subsystems becomes impossible if at least one of them is correlated with a third subsystem.

The kinematics of correlations, as treated in the framework of axiomatic local quantum-field theory, was born in 1985 in the work of Summers and Werner[227], taking shape in 1987 by the works of the same authors[230] and Landau[160, 162]. The commuting algebra $\mathcal{A}_i$ we have used is that of local observables which correspond to spacelike separated regions in Minkowski spacetime. This in itself is not new[51, 227, 147] but the results of [51, 147] are independent of the notions and methods specific to field theory, such as Lorentz invariance, vacuum state, spectrality condition, mass gap, Reeh-Schlieder theorem, etc., and thus are related to the more general class we have already considered. The main results of [227], depending on field theory, are obtained for free fields, and are related to a more particular class which will be treated separately.

The Schlieder property (see, for example, [148, pp. 27, 64, 77, and 80] combined with equality (1.5) allows us to prove[160] the equality

$$\|C\|^2 = 4 + \|[A_{11}, A_{12}]\| \cdot \|[A_{21}, A_{22}]\|$$

The relation $R = \|C\|$ can always be obtained by a proper choice of the state. Further, in any noncommutative $\mathcal{C}^*$ algebra, A_{11} and A_{12} can be selected[160] so that one has $\|[A_{11}, A_{12}]\| = 2$. In that manner, the proper choice of state and observables can give the equality $R = 2\sqrt{2}$ (see also [22, Sect. 5]) whatever the sizes and shapes of the regions and their separation[160]. As to the vacuum state, taking into account its cluster properties, one will obtain[227] the inequality $|R| \leq 2+\varepsilon$, where $\varepsilon \to 0$ with the increased spacing of the regions. In the presence of the mass gap, ε is

[5] We are thankful to him for this communication.

exponentially small when the spacing is large compared to the Compton wavelength regardless of the choice of observables. When there is no mass gap, ε decreases as the inverse square of the separation, the coefficient being dependent on the choice of the observables. If, in contrast, we draw together the regions, it is natural to expect a reinforcement of some correlations in the vacuum (and, what is more, in any non-pathological) state. The case of complementary wedges (see, for example, [148, pp. 36, 227, and 241]) whose spacing is equal to zero (there are common boundary points) is examined in detail. In that case the presence of correlations is necessary (any factorized state is not normal; see [148, p. 75]). And now the equality $R = 2\sqrt{2}$ can be obtained by a proper choice of the observables rather than of the state[230]. For regions separated by a positive spacing, one notes[230] that the "funnel property" (see [148, pp. 57, 74, and 266]) establishes the existence of factorized states—for which, of course, we have $|R| \le 2$. On the other hand, using the Reeh-Schlieder property (see, for example, [148, pp. 30 and 34]) one can prepare almost any state from the vacuum by means of the selection of some (rare) local events. Due to this, the relation $R \approx 2\sqrt{2}$ can be "simulated" in a vacuum too (and in any of a wide class of states), but now for conditional probabilities[162] (the condition itself may be very rare).

The kinematics of correlations embedded in the free-field theory is distinguished by the use of canonical commutation or anticommutation relations (CCR or CAR) for the field operators. Summers and Werner[227, 228, 229] have shown that $R \approx 2\sqrt{2}$ in the vacuum state if proper observables are selected from local algebras of free boson or fermion fields (massive or massless), and two regions in the spacetime are selected as mutually complementary wedges. It is obvious that this is a particular case of the result considered above from [230]. The vacuum state of the free boson field itself proved to be similar to the state considered by Einstein, Podolsky, and Rosen[70]. One can indeed select four observables[229], linear in the field operators, P_1, Q_1 localized in the first region, and P_2, Q_2 in the second one, such that

$$\begin{aligned} [Q_1, P_1] &= i \\ [Q_2, P_2] &= i \\ \langle (P_1 + P_2)^2 \rangle \langle (Q_1 - Q_2)^2 \rangle &\ll h^2 \end{aligned}$$

(these are mean values in the vacuum state). After that, one is able to construct[229] observables A_{ij}, giving $R \approx 2\sqrt{2}$. However, those observables are no longer linear in the field operators. They have neither a simple algebraic nature nor a simple operational sense. A trick of [229] employs a good state but poor observables. There is another trick

[147, p. 444] with good observables but a poor state. The application of the discontinuous sign function to the CCR operators P_i and Q_i gives $\|[\mathrm{sign}(Q_i), \mathrm{sign}(P_i)]\| = 2$, from which one deduces, through equality (1.5), that $R = 2\sqrt{2}$ for a particular state. The latter is explicitly written in [147] but has no simple operational sense [as opposed to the observables $\mathrm{sign}(Q_i)$ and $\mathrm{sign}(P_i)$]. But if good observables are employed (the operators obeying canonical commutation relations, and their spectral projections) combined with a good state—a Gaussian one, in particular a vacuum state, a coherent one (including "squeezed" ones), or a Gibbs state for the quadratic Hamiltonians—then one would have $|R| \le 2$ (see [18, 161]).

The kinematics of correlations lying beyond the scope of quantum theory was introduced in 1985 by the present authors[147] and Rastall[204] (see also [209]). Returning to (1.1), one can say that here the right-hand side, which depended on the pecularities of quantum theory, is dropped; the left-hand side is treated as before; and the question is raised of how we may express in these terms the causality principle forbidding faster-than-light signals. Rastall[204] has confined himself to the case when each of the indices i, j, k, takes on only two values, indicating that the equality $R = 4$ can be satisfied without a causality violation. In the quantum theory this is forbidden by inequality (1.3) (but allowed for triple correlations [189, 93]). From (1.2) it is easy to see that $R = 4$ in only one case: i.e., when $\langle A_{ij} \rangle = 0$ and there are perfect correlations: $A_{11} = A_{21}, A_{11} = A_{22}, A_{12} = A_{21}, A_{12} = -A_{22}$. There is no inconsistency here because the observables A_{i1}, A_{i2} are complementary ones unlike A_{1j_1}, A_{2j_2} which are compatible (see [204] and Section 4 of [158]). Simultaneously with Rastall, $R = 4$ was examined by the present authors[147, p. 451] along with a number of examples which illustrate how we have axiomatized the causality principle covering any number of values of the indices i, j, k and more general situations. Summers and Werner[228] have treated pair correlations beyond the quantum scope from the angle of observable algebras more general than C^* algebras. If at least one of the two algebras is C^*, then inequality (1.3) remains valid[228], and the conditions for its reducing to the equality are close to those obtained[235] in the quantum domain.

The kinematics of correlations treated as embedded in classical theory was introduced, of course, by Bell[22] in 1964. The bibliography on this question is vast. There are several reviews: Home and Selleri[112], De Baere[60], Spassky and Moskovsky[223], Grib[95], Clauser and Shimony[47], and others. We confine ourselves to remarks. In our view, the question is, as before, that posed near (1.1); however, now the observable algebra is assumed to be commutative. Mathematically this is a great simplification. In the quantum case, as we saw, a substantial task which we have formulated with reference to (1.1) is completely solvable only in the simplest situations. In the classical case this task is solvable, though not in the form of a general formula but in an algorithm giving a precise solution for a finite set of i, j, k values in a finite number of steps. This was recognized in l981 by Froissart[78], who has implemented the algorithm by computer for some situations (see also [84, 82, 173]). In the simplest situation, where i, j, k take on two values each, the solution is given by the inequality $|R| \leq 2$ (obtained in [48] and known as CHSH or Bell-CHSH inequality), together with three similar inequalities obtained from it with the use of obvious symmetries. We see now how expression (1.2) is distinguished among all other linear combinations of four correlations. Geometrically, the equality $R = 2$ is the equation of a face of the corresponding polyhedron. Therefore, the use of the quantity R is reasonable in such classical or quasi-classical cases when $|R| \leq 2$ or $|R| \leq 2+\varepsilon$. In the quantum case, instead of a polyhedron there occurs a convex body, the equality $R = 2\sqrt{2}$ being that of the plane tangential to this body at one of the points of its surface. From the quantum point of view, inequality (1.3) is only one of a continuum of linear inequalities. This fact is taken into account in [51, 235, 158], though the same cannot be said for [227, 228] and [160, 161, 162, 229, 230]. Note once more that in the classical case we have $R_3 \leq 2$ [cf. (1.7)] [189]. Finally, there is a theorem on the comparison of quantum and classical correlations, obtained by one of us[235] for the case where i takes on two values, and j, k as many as one wants. If the matrix M is that of quantum correlations, then the matrix θM is that of classical correlations provided that the positive number θ satisfies the condition $\theta K_G \leq 1$, where K_G is Grothendieck's constant (the latter being an exact constant for the mentioned theorem). This constant has been studied by mathematicians since 1956, but as yet it is only known that $K_G \approx 1.68 \pm 0.11$. The role that Grothendieck's constant plays in correlation matrices of any size is the same role that $\sqrt{2}$ plays in 2×2 correlation matrices. This surprizing physical meaning of K_G encourages mathematicians to search for an example where quantum correlations exceed classical ones by more than $\sqrt{2}$ times. It appears unexpectedly hard. Fishburn and Reeds[77] found a matrix of size 20×20 giving the ratio $1.428 > \sqrt{2}$. Note that nothing like this is known for triple correlations.

To complete the review on kinematics of correlations, we want to emphasize that it utilizes several commuting algebras, which correspond to several spacelike separated regions in spacetime; we hope that this kinematics is just a prologue to a more substantive multiple-time theory, i.e., to the dynamics of correlations.

In this more general theory we have no initial formula so simple, general, and indisputable as is (1.1) for the kinematics of correlations. The latter can be confined to the interaction of a quantum system with observers who have to select what observables are measured but cannot affect the system. The inevitable influence of measurements on the quantum system is not caught by the kinematics of correlations, which does not take into account the state of the system upon making a complex of synchronous measurements. As an alternative to the picture of the system and observers guided by kinematics, dynamics pictures a system and experimenters whose interaction with the system is of a bilateral nature. In this case, both the system and the experimenters are distributed in spacetime and interact locally. One may speak of a collection of local experimenters or a series of local experiments. But this is always a terminological question. Of course, the experimenters are described classically. While the concept of observer is traditional in physical theory, the same cannot be said of an experimenter provided with free will.[6] This innovation of principle provoked serious discussions in the 1970s (republished as [23]), and has been discussed again more recently (see [161, Sect. 5.2] and [193, 187, 31]).

It is possible that (1.1) should be interpreted as dynamically guided if the measuring equipment is included in the system in question. (The permissibility of such an approach will be discussed in the next section). Let the local experimenter i be thought of as pressing key j_i on his control panel. After some time he will perceive that indicator lamp k_i has fired. Previously, one would have said that pressing the key had given rise to measuring a corresponding observable. Upon including the equipment in the system one would say otherwise: by pressing on the key, the experimenter has affected the system. By looking at the lamps, the experimenter has taken a measurement. The observable is the number of the lamp that fired; the index j disappears into the notation of the observable A_{ij}.

Now it is clear how the multiple-time generalization of the left-hand side of (1.1) can be given. The index i must run through a partially ordered set which is interpreted as a set of local chronologically ordered experiments. The index j_i sets the influence upon the system, exerted in the ith local experiment; the index k_i, the system's response to this influence (and to others, as far as the causality principle allows), measured in the ith local experiment. However, the further the theory is developed, the more inconvenient the notation inherited from a simpler situation becomes. Let us go over to the notation proposed by us in [147]. In [147] there is in fact a much more general formalism permitting a continuous spectrum of

[6]Interestingly, N. Bohr, while a student, looked for a mathematical solution of the free-will problem [54, p. 13].

values for i, j, k, nonlocal (e.g., loop) observables, etc. But such a general formalism requires that the reader possess a previous familiarity with such elements of modern probability theory as flows of σ-algebras and regular conditional probabilities. Taking into account that physicists rarely make use of that language (excluding, perhaps, specialists in constructive quantum field theory), we will formulate less general definitions.

Let a finite set S of "world points" be given which is partially ordered by the relations $s_1 < s_2$, meaning that it is possible to send a signal from s_1 to s_2. Further, let two finite sets $\Omega_0(s)$ and $\Omega_1(s)$ be given for each $s \in S$. $\Omega_0(s)$ is interpreted as a set of possible local influences; $\Omega_1(s)$, that of possible local responses. The *integral influence* is the function ω_0 on S such that $\omega_0(s) \in \Omega_0(s)$ for each s. Similarly, define the *integral response* ω_1. In addition, we will need a discrete analog of the spacelike surface dividing the future from the past. To this end, let us define an ideal as a subset t of the set S, having the following property: if $s_1 < s_2$ and $s_2 \in t$, then $s_1 \in t$. It is understood that t is interpreted as a region of the past. By restricting one's attention to the points in t, one defines in an obvious way the notion of a "t-integral influence" and a "t-integral response" (denote them, respectively, as ω_0^t and ω_1^t).

We define a *stochastic behavior* as a functional $p(\omega_0, \omega_1)$ of the integral influence ω_0 and integral response ω_1 such that: (a) as a function of ω_1 at any fixed ω_0, it yields a probability distribution, i.e.,

$$p(\omega_0, \omega_1) \geq 0; \qquad \sum_{\omega_1} p(\omega_0, \omega_1) = 1$$

(b) for any ideal t and any t-integral response ω_1^t, one has

$$p(\omega_0, \omega_1^t) = p(\omega_0^t, \omega_1^t)$$

Here $p(\omega_0, \omega_1^t) = \sum p(\omega_0, \omega_1)$ and the sum is performed over all integral responses ω_1 coinciding with ω_1^t at the points from t; in other words, the summation is performed over the variables $\omega_1(s)$ for all s outside of t. The origin of condition (b) lies in the fact that the mentioned sum, regarded as a function of ω_0, is really independent of $\omega_0(s)$ for s outside of t.

Condition (b) expresses the principle of causality: the probabilities of the responses in the past do not depend on the influences in the future. In this manner, the definition of stochastic behavior gives an axiomatic foundation for formulating problems beyond the scope of quantum theory as well. The scope of a classical theory can be axiomatized as follows.

A *deterministic behavior* is a stochastic one which satisfies the "condition of determinism," i.e., for any ω_0 and ω_1, the number $p(\omega_0, \omega_1)$ is equal either to zero or one.

A *hidden deterministic behavior* is a convex combination of the deterministic behaviors

$$p(\omega_0, \omega_1) = \sum c_k p_k(\omega_0, \omega_1)$$

each p_k being a deterministic behavior, and the coefficients c_k such that $c_k \geq 0$, $\sum c_k = 1$.

With regard to mathematics, the situation here is no more complicated than in the kinematics of correlations. Froissart's considerations [78] remains valid. The set of all stochastic behaviors is a convex polyhedron in an appropriate finite-dimensional space. The equations of its faces are known; therefore, a finite number of algorithmic steps give its vertex points, which represent extremal stochastic behaviors. The set of all hidden deterministic behaviors is another convex polyhedron (lying inside the first one). Its vertices are known; therefore, a finite set of algorithmic steps gives the equations of its faces, which are nothing but multiple-time Bell inequalities. To return to the kinematics of correlations, it is necessary simply to take S with the trivial ordering when there is no one pair s_1, s_2 such that $s_1 < s_2$. If in so doing S consists of two points only, one will obtain $|R| \leq 4$ once again for the stochastic behaviors, and $|R| \leq 2$ for the hidden deterministic ones [R being defined by (1.2)]. It is natural to expect that $|R| \leq 2\sqrt{2}$ for the quantum behaviors if this notion is defined in the right way. One may give the definition of a *projection-valued behavior* $P(\omega_0, \omega_1)$, which literally repeats that of the stochastic behavior with no other difference than that $P(\omega_0, \omega_1)$, at the given ω_0, ω_1, is not a number but a Hermitian projection in the Hilbert space. Given a projection-valued behavior and a quantum state, one obtains a stochastic behavior,

$$p(\omega_0, \omega_1) = \langle P(\omega_0, \omega_1) \rangle \tag{1.9}$$

We can see that this formula, if we take into account the inclusion of the equipment in the quantum system, shows that quantum theory may describe only those stochastic behaviors p which permit representation (1.9) via some P and some state. In this respect, the presentation in the form (1.9) is a necessary quantum condition. Is it a sufficient one? We do not know. A detailed discussion of this question is given in [147]; see also [241]. In the kinematics of correlations, (1.9) turns into (1.1), the necessary condition proving to be sufficient. Note also that (1.9) does not leave the class of stochastic behaviors. Quantum theory does not permit faster-than-light signals. This has been shown many times. A number of references (and one more proof) can be found in [205]. However, an exhaustive analysis was given by Schlieder[213] in 1968 (see also [99, 105]).

The study of the dynamics of correlations in the framework of a general quantum theory was begun in [51] and continued in [147]. Two equivalent forms of the necessary quantum condition was given (one as above, another in terms similar to a scattering theory). A further analysis of various definitions of this kind is given in the work of Vershik and Tsirelson[241] intended for mathematicians. If a conjecture made in [241] is proved true, a way will have been opened toward "boson Bell-type inequalities" valid for any system of interacting Bose fields but violated by Fermi fields!

All of the above problem settings are expressed in terms of probabilities. If we refer directly to the Hilbert space without the mediation of probabilities, we can relate to the kinematics of correlations the application of the Schmidt canonical form

$$\psi = \sum_k \sqrt{p_k}\phi_k \otimes \Phi_k \tag{1.10}$$

to express an arbitrary state vector ψ of a composite system via orthogonal sets of state vectors $\{\phi_k\}$, $\{\Phi_k\}$ of two of its subsystems (see [242, Sect. 6.2] and [79, 243]). In contrast to the trivial expansion of the form $\sum \lambda_{kl}\phi_k \otimes \Phi_l$, in (1.10) the vector ψ determines uniquely the numbers $p_0 \geq p_1 \geq \cdots \geq 0$, and if all p_k are different, then ϕ_k, Φ_k are determined uniquely, as well. A connection with Bell inequalities is pointed out qualitatively by Home and Selleri[112, sect. 2.1.4] and quantitatively by Tsirelson[234]:

$$R \geq 2p_0 + 2\sqrt{2}(1 - p_0)$$

for some A_{kl} and some state [see (1.2)]. A change in ψ according to the Schrödinger equation gives rise to changes in ϕ_k, Φ_k, by the proper equations of motion found in Kubler and Zeh's work[154], which thus may be related to the dynamics of correlations. One more branch of this discipline, i.e., a dissipative dynamics of correlations, is presented in Section 4.

2 Universal Wave Function

Historical successes of classical physics have called into being an assumption on the universal applicability of the most general features of the classical description. Subjected to the impact of quantum theory, this assumption has undergone a number of refinements now to be crystallized in the form of a hypothesis on hidden deterministic behavior (see Section 1) according to which only behavior of the type noted above can be observed by experience. Experimental results leave practically no doubt (see Section 7) that this hypothesis is false. The successes of quantum theory have given rise to a similar assumption that the quantum description principles possess universal applicability. Neither Bohr nor Heisenberg initially regarded quantum theory as a universal physical theory; however, it proved to be much more viable than they expected at the time, as Weizsacker writes [247, p. 285].

What does it mean to the experimenter? How does one test universal applicability of quantum theory? We know of two relevant trends (see Section 7): macroscopic quantum phenomena and quantum Bell-type inequalities.

A negative result of some test on macroscopic quantum coherence (that is, absence of such coherence when quantum theory predicts it) would reveal applicability of the classical theory in some domain, and apparently would result in establishing some hybrid quantum/classical theory, such as spontaneous measurement theories (see Section 5).

A negative result of some test on quantum Bell-type inequalities (that is, violation of such inequality when quantum theory predicts its fulfillment) would reveal a restriction on applicability of quantum theory, and likewise of any hybrid quantum/classical theory of the above-mentioned type (see Section 5).

Thus, in the first case the position of classical theory would become stronger at the expense of the position of quantum theory. In the second case, by contrast, the position of any possible "quantum/classical alliance" would became weaker, regardless of alignment of forces within the alliance.

The idea of universal applicability of quantum theory led to the concept of "universal wave function," which describes all systems at a single level (without separating them into objects and instruments), and never undergoes reduction. Indeed, existence of this function may only be postulated; it fails to be well founded via the usual arguments concerning preparations, measurements, ensembles, probabilities, states, observables, and so on; see Woo [252]. However, neither can the usual "microscopic" wave function be well founded in this way. It is enough to have heuristic indications of an appropriate formalism and successful accounting for facts on the basis of this formalism. As long as no experiment violates a quantum Bell-type inequality, their universal validity may be considered as a fact calling for an explanation, and the universal wave function as a concept providing such an explanation. One may say that the universal wave function is not essential here, the usual "Copenhagen" one being enough. However, as Stapp [226] stresses, the Copenhagen interpretation allows one to apply quantum theory, strictly speaking, only under special conditions where some part of the universe is first separated from its surroundings, left (almost) isolated during some finite time, and then reunited with it. In chemistry, especially in biochemistry, there often appear situations where microscopic degrees of freedom are continuously included in long chains of (not too weak) interactions which eventually span the gap up to the macroscopic world.

If quantum theory is indeed applicable only to temporarily isolated systems, then, what theory should to be applied to other, nonisolated systems? If a reduction in isolation may indeed result in a deviation from quantum laws, then, in what direction? Possible answers follow: (1) toward an essentially new, yet unknown theory; (2) toward a hybrid quantum/classical theory; (3) no deviation; the quantum theory is always applicable, if formulated properly. Variant (1) will be applicable in the event of violation of a quantum Bell-type inequality in some "physico-chemical," "physico-chemico-biological," or other experiment. Variant (2) will apply in the event of a negative result of some test on macroscopic quantum coherence. As long as this is not the case, variant (3) is preferable provided there is a formulation of quantum theory not restricted to temporarily isolated systems. The hope of finding such a formulation stimulates a number of works, including Stapp's and ours. Before discussing available proposals, let us answer one objection to the universal wave function.

Macroscopic systems, as a rule, fail to be isolated to the extent required for their quantum description. Upon including one macroscopic device into the universal wave function, one is forced to include in it the entire Universe. Specialists on quantum cosmology approve of this (see, e.g., [116, 256, 33, 102]), but the same cannot be said of other physicists. We think that to use the universal wave function it is necessary to use an astronomical rather than a cosmological scale. This conclusion is related to the speed of light. In the framework of the Copenhagen interpretation the spacetime domain "occupied" by a quantum system can be regarded as a cylinder. Preparation corresponds to the lower base, measurement to the upper base of this cylinder, and isolation from the environment to the lateral surface. Permitting curvilinear spacelike surfaces as bases, one can get rid of the lateral surface entirely, and thus the question of isolation itself! The possibility of such a description "curvilinear in time" is well known in quantum field theory.[7]

Now we proceed to well-known interpretations of universal wave function.

First, in 1952, D. Bohm[26] set up an interpretation he named "objective" or "causal." In the one-particle case it conforms to earlier ideas

[7] Generally speaking, the initial state in the given spatial region is of course correlated with the state outside the region. This is, however, not a great obstacle. One must simply consider the dynamics of the relative state inside the region when fixing a state outside it.

of de Broglie on the so-called pilot wave. There is, however, substantial interest in the many-particle closed system, possibly including in itself macroscopic instruments. Dynamics is described by two mathematical objects: the wave function (obeying a conventional Schrödinger equation) and the "classical state," i.e., a set of coordinates and velocities of all particles (deterministic equations of motion are postulated for them which are dependent on the wave function via a non-local "quantum potential"). This interpretation can be rebuilt for the field theory instead of mechanics. Its present-day state is presented in [17, 27, 210].

Second, in 1957 Everett[74], supported by Wheeler[249], set up a "relative state" interpretation (see also [63, 258, 4, 61, 255, 257]). The state of a closed system (including the macroscopic apparatus itself) is described by a wave function obeying the conventional Schrödinger equation (or, maybe, its geometrodynamical generalization). It is of major interest for this interpretation to consider the case when the system contains subsystems capable of storing information. In that case the wave function is dynamically decomposed into orthogonal summands which evolve independently of each other; phase relations between them do not manifest themselves. The relative state interpretation is not the same as the "many worlds" interpretation (see, e.g., [63, 250]). Ideas close to the relative state concept have been expressed [185, 79] much earlier than in 1957 and much later than Everett [150].

Finally, in 1967, Pearle[192] pointed out that the Everett-Wheeler approach rests upon two theses which may be used separately. The first thesis: instruments must (not only may) always be included in a state vector; this enables the reduction to be eliminated from dynamics. The second thesis is adopted from the Copenhagen interpretation: a state vector describes an individual physical system completely. An alternative thesis was supported by Einstein[72], Mandelstam[178], and others: a state vector describes an ensemble of identically prepared systems. Uniting this with the first thesis, Pearle obtained a "statistical interpretation." For its further development see Ballentine[13, 12]. For various treatments of the notion "ensemble," see [28, 6].

It is of no particular importance to us whether the wave function describes an individual system or an ensemble, since we do not know how to discriminate between these two principles by a feasible experiment (now or in the foreseeable future), if both principles are properly applied to the universal wave function. Note common features of the above interpretations:

(a) quantum dynamics is accepted as it is, without any man-made deviation toward classical dynamics (see Section 5 about such deviations);

(b) there are no classically described physical systems, but there is a classical reality.

Let us consider the relationship between quantum dynamics and classical reality from a formal point of view. In the relative state interpretation, classical reality arises from classical events. Each of these events consists of decomposing the state vector into a sum of some "fragment" vectors which are not only orthogonal but "hereditarily orthogonal," in the sense that a further evolution of various summands (with a further decomposition into smaller fragments) takes place in various subspaces orthogonal to each other. It is convenient to regard a set of fragments as a phase space of a Markov process. When the fragment is decomposed further, a random point jumps to one of the "offspring fragments," and the probabilities are, naturally, set to be the squared vector length. The path of such a random process is interpreted as the classical history. The latter is discrete in time (from event to event); this fact results from the assumed discreteness of information storing systems, and is not a matter of principle. It is rather a matter of principle that the very existence of classical reality and its character depends on existence and properties of these storage systems.

The "objective" interpretation uses, at first sight, quite another mathematics. However, just now, in passing from mechanics to field theory, it loses its initial obviousness, the abstract "skeleton" of the applied constructions becoming clearer in return (and the arbitrariness contained in them becomes more noticeable). At each instant the state vector is presented in the proper way in the form of a sum of orthogonal summands, i.e. expanded in the eigenvectors of a singled-out complete system of commuting operators. There is a probability distribution on the set of these summands. Then, based upon the constructed single-time distributions, one selects a way of constructing the many-time distributions. This leads to a random process. Finally, one randomly given path of that random process is proclaimed to give an objective description for the evolution of the physical world.

The resemblance to the Everett interpretation is evident in such a description. The difference is, first of all, the absence of a hereditary orthogonality. Fragments are not only "decomposed" but "recombined," and rather intensively. A deeper difference follows. In the Everett interpretation classical reality rests on dynamic properties of the quantum system (its capability of receiving and retaining information, no matter in what form and in what medium). The "objective" interpretation rests on kinematic structures. Namely, it has postulated for mechanics an exclusive role of coordinate observables, and for field theory an exclusive role of infinitesimally localized observables. The contrast is striking for the vacuum state. According to the Everett interpretation, no classical events occur in vacuum (as there is no accumulation of information). The "objective" interpretation ascribes to vacuum a rich-in-contents classical reality ("hard boiling"). We cannot make exact statement about this, since the field version of this interpretation is weakly developed, and the "usual" quantum field theory is far from being rigorous. However, it is clear that observables, localized in infinitesimal domains, certainly have infinitely large fluctuations resulting, at best, in very complicated classical configurations. The "objective" interpretation takes as a basis a structure rather than a function ("anatomy rather than physiology"), namely the spatial arrangement of ("bare") elementary particles. This makes it possible to introduce "classical reality" with enviable clearness, but at the cost of imagining the unobservable microscopic classical reality. We do not follow this trend.

There is a school of opinion that Everett simply introduced a bizarre terminology, calling reality what was formerly called the collection of possibilities. On the other hand, his interpretation is often called "many worlds," with the understanding that the coexistence of many worlds is in the same sense as when one says that subsystems of one physical system coexist. We have to emphasize that the construction of the universal wave function is expressed, by Everett, in terms of two mathematical operations: tensor multiplication and superposition (addition with complex coefficients). The meaning of the tensor multiplication is rather clear: it describes the coexistence of subsystems. The meaning of the superposition which joins alternatives ("worlds") is quite different and substantially nonclassical. It approaches the classical idea on "possibilities" insofar as the interference between the alternatives can be neglected.

Everett's work[74], as was rightly noted by the author himself, is a meta-theory rather than a physical theory. It is no more (however, no less) than a research program intended for building a new interpretation. The scenario of state vector decomposition into "fragments" has been obtained on an *a priori* assumption that quantum theory is actually capable of describing, in such a way, the physical world together with macroscopic instruments and even observers. We appreciate Everett's work as a call for reconstructing the classical world from quantum dynamics, on the basis of an analysis of dynamic characteristics of big quantum systems. A number of works on this trend, from Everett until today, will be discussed in Section 3.

All interpretations of the universal wave function correlate the classical description with the quantum one in a quite new manner against the Copenhagen interpretation. The last requires for each case to divide the relevant physical systems into those described as quantum and those described (for this case) as classical. A joint application of both description modes to one system is senseless in the Copenhagen framework. By contrast, the universal wave function is always used for a joint quantum/classical description (not to be confused with a hybrid dynamics in the sense of Section 5), the quantum description being primary, and dynamically independent of the classical one, the latter being now a superstructure over the former. There is no point in opposing them in the spirit of Copenhagen. To explain what we mean by "superstructure", let us list several other examples of it (see also Clarke[46]). (a) A representation of a group or an algebra in a Hilbert space is a superstructure over this Hilbert space. When someone says that a system is described by a representation of the commutation relation $PQ - QP = -ih$, no one objects that only one thing may by fundamental, either the Hilbert space, or the abstract algebra generated by P, Q. (b) A Riemannian metric is a superstructure over a smooth manifold, that turns it into a Riemann space. Also the smooth manifold is a topological space with an overstructure, namely, the smooth structure. And the topological space, in its turn, is a set provided with a topology. When someone says that space-time is described with a Riemann space, no one treats it as several competing fundamental descriptions.

So, we strive to reconstruct the classical world as a superstructure over quantum dynamics.

3 Reconstruction of the Classical World: Approaches to the Problem

Is there a criterion for applying the classical description within the quantum theory? If so, then one can reconstruct a classically described macroscopic world embedded in a more fundamental quantum description, thereby supporting the idea of the universal wave function. If not, the departure point of the Copenhagen interpretation remains valid: quantum theory must be constructed on a preliminary classical theory not only historically but logically.

The Copenhagen interpretation takes classical physics as a given, but limits its use to situations where all representative quantities having the dimensions of action are large compared to Planck's constant. It is interesting that as early as half a century ago Teller questioned Bohr as to why the accepted classical concepts could not be replaced by others. Developing this idea, Teller assumed that irreversibility is the sole property of the macroscopic apparatus important for quantum theory. Bohr, however, did not believe it was correct to make first principles dependent on a "particular theory such as thermodynamics." We have discovered this from the paper of Weizsacker[248], who supported Teller on this point. It looks as if Teller anticipated two ideas which have become indispensable for the universal wave function to be interpreted. First, the meaning of the term "classical description" considered in the context of the quantum theory foundations can differ from that of the same term used in the history of physics. Second, the most important element of the classical description for a quantum viewpoint is that of thermodynamic irreversibility.[8]

"Can the quantum theory stand on its own feet?" It is in these terms that the problem often appears in the discussions which have go on for decades, and whose participants included the great physicists, and makers of quantum theory (Einstein, Bohr, Schrödinger, de Broglie). This problem is resurfacing with new strength at present. The quantum description of macroscopic systems which are used as instruments attracts great attention from the "optimists," who have made a number of important observations and stated more than once that a positive solution of the problem has been found. However, all the suggested solutions were met with criticism from the "pessimists," who emphasize a failure in resolving the problem so posed. We enumerate some ideas of this kind.

Wakita[244] pointed out that a macroscopic collective degree of freedom, as a rule, strongly influences the numerous microscopic degrees of freedom coupled to it; this causes a fast propagation of quantum correlations, that is, an "entanglement of the wave function." Dynamical "disentanglement" is prevented by thermodynamical irreversibility. At the same time, the macroscopic interactions are described by the observables which are decomposed into sums of summands each of which depends on few degrees of freedom. It is therefore very difficult to distinguish between a strongly "entangled" wave function and a mixture. The macroscopic information can be said to exist in many "copies," some small part being transferred usually by the macroscopic interaction.

Daneri, Loinger, and Prosperi[52, 53] have analyzed a number of typical ways of taking quantum measurements to demonstrate that in each case the device transits from a metastable state to a stable one—the parameters of the latter containing the measured result. The macroscopic interaction of the device with its surroundings is described by observables that do not mix together various stable states; otherwise the regular thermodynamic behavior of the device would be violated. Jauch, Wigner, and Yanase[115] objected that the limitation of the class of observables for a device does not result from the first principles of quantum theory (see also [114, 208, 34, 175]). Both in [53] and [115] it is noted that the use of the master equations is a promising way to describe the quan-

[8]The problem of irreversible statistical physics (and thermodynamics) foundation in the scope of quantum theory is discussed in Section 6.

tum/classical correspondence (see also [20, p. 177]). The simplest pattern of ideal measurement[242] was generalized by the notion of "operation" (see Haag and Kastler[99], Schlieder[213], Hellwig and Kraus[106, 105], and Kraus[151]); for the continuous case, both in time and in the spectrum of measurement outcomes, see Davies[56] and Davies and Lewis[57]; for further investigations we refer the reader to the survey [35]; see also [42] and [172].[9]

Hepp[107] has demonstrated the applicability of the classical description in the situation where the information from the readings of the instruments is propagated without limit in the environment, while the macroscopic interaction of instruments with other (secondary) ones is described by local observables. Bell[19] objected that this limitation for the class of observables does not result from first principles, all the more so because information can propagate only a finite distance in a finite time.

The "delocalization" of quantum correlations is undoubtedly important for quantum theory, throwing light on a number of questions. Why are molecules of ammonia usually observed in states of definite parity and those of sugar in states of a definite chirality? Why are microscopic oscillators usually observed in states of definite energy and macroscopic ones in states more like coherent states? Why does one of the two correlated particles in the EPR thought experiment contain information of both a coordinate and momentum of the other particle while the instrument, upon interacting with the object, possesses the information only on one definite observable? Why are macroscopic bodies usually observed in states having a slight dispersion of coordinate and momentum, in spite of the fact that a non-well-integrable dynamics, as a rule, increases this dispersion exponentially? Why do the coordinates and momenta of macroscopic bodies obey the classical equations of motion which are often treated as asymptotic consequences of the Schrödinger equation, while the Schrödinger equation itself cannot be satisfied by such objects even by a crude approximation, due to the strong (on a quantum scale) interaction with the environment? How do these facts affect the limiting process $h \to 0$? Finally, why is the spacetime metric observed as classically definite and obeying the classical Einstein equation? These and related questions are considered in [259, 116, 117, 146], and [260, 15, 266, 265, 118, 119, 263, 262].

Let us interrupt the enumeration of ideas in order to analyze the origin of the discord between "optimists" and "pessimists". To begin

[9] "The problem is that we are implicitly trying to apply the quantum measurement nostrums developed in the 20's and 30's ... to experiments of a type only feasible in the 90's (if then!) where we seriously wonder about the effects of quantum mechanics on *macroscopic* bodies"[172].

with, note that the quantitative difference between the microworld and the macroworld is great but finite. There are quite a few opportunities for reconstructing a classical world from a quantum one through a limiting procedure, when a parameter of a quantum system tends to infinity [244, 107, 177]; thereby, effectively classical degrees of freedom (or events) emerge. It can be said that classical systems are infinite (in a special sense) quantum systems. Thus, finite systems are described via their relation to imaginary infinite systems. Such a description is considered to give a satisfactory approximation for the relation of small (microscopic) systems to large (macroscopic) ones.

A serious flaw in the above approach lies in the fact that the limiting procedure makes the picture qualitatively more scanty, sometimes leading away from essential points. Thus, classical dynamics transform a point of the phase space to a point, and it may be treated as the simplest asymptotics for the movement of a localized wave packet. However, usually trajectories diverge exponentially, and the considered time interval exceeds the time of diverging. In the case the above asymptotics needs the unrealistic assumption that parameters of the system are not only large, but also exponentially large in comparison to Planck's constant. A realistic analysis of large finite systems has led to the realization of the principal role of delocalization of quantum correlations. Macroscopic bodies are not only large, but also essentially coupled with their environment. The present criticism concerns also the Copenhagen interpretation describing finite quantum systems via their relation to classical ones. One may object that classically described macroscopic bodies are not considered infinitely massive; their coordinates and momenta are acknowledged to be defined up to small uncertainties obeying Heisenberg's relation. However the point is that these uncertainties are assumed necessarily small just by virtue of the classical nature of the body, and thereby the problem of wave packet spreading is abandoned *a priori.*

So we consider it unsatisfactory to substitute infinite (or necessarily classical) systems for macroscopic ones. In this regard we agree with the "pessimists." Even so we do not reject the arguments of the "optimists." Let us explain how it is possible, starting with a digression.

In olden times the question "where are we in the Universe?" admitted a short, exact, and clear answer: at the center of the Universe, that is, at the point, around which the Universe rotates. Nowadays, the answer is necessarily lengthy and vague; it involves a lot of details about specific galaxies, etc. If someone considered this situation a great loss, he would readily find a few opportunities to embroider the picture of reality (the world view). For example: let all the Universe be stitched with coordinate lines (woven from ether) that are unobservable to us, but for nature itself they perform important duties: they fix the positions of all (classical) bodies! (This is a parody on the "objective" interpretation). Another example: let Newton's equations of motion be supplemented by a new term describing a "spontaneous movement," always directed at a certain point (the center of the Universe), but too slow to be detected in our time. (This is a parody on "spontaneous measurement" theories; see Section 5).

It is easy to speak ironically of concepts given up long ago. It is hard to adopt the idea that our classical reality is singled out not by nature in itself, but by our specific position within nature. However, it is this idea that seems to be the next step away from our anthropocentrism. We acknowledge that another observer may disagree with us not only on the meaning of "up" and "down", but even on the meaning of "classical reality".[10] One may object that such a different observer would necessarily be beyond the reach of our communication, so it is pointless to consider him an observer. We answer: but an observer within a black hole is out of reach of our communication, as well.

So, we agree with D'Espagnat[58] that the origin of the disagreement between the "optimists" and the "pessimists" is the fact that the "optimists" investigate the emergence of classical reality relative to a class of observers, whereas the "pessimists" acknowledge only absolute (independent) classical reality.

As far as we know, the first "optimist" was Teller, who proposed thermodynamically irreversible systems for the role of the observers, and the first "pessimist" was Bohr, who objected that the notion of classical reality is more fundamental than thermodynamics. We consider works of the "optimists" as progress rather than a collection of mistakes and misunderstandings, and perceive nothing reprehensible in using statistical physics for the foundations of quantum theory, as well as for other areas, bearing in mind that, thereby, these foundations became dependent on problems of the foundation of statistical physics. Also, we perceive, in principle, nothing reprehensible in taking account of the observer's size, lifetime and so on, provided that it is really essential.

In classical theory an exact description of the observed object goes together well with an extremely idealized one of the observer, owing to the concept of an undisturbing measurement. Acknowledging the influence of the observer on the object to be inevitable, quantum theory has made itself responsible for describing a real (nonideal) observer. In practice,

[10] This is not a modern idea. Remember the Wigner friend paradox [251]. Moreover, the Schrödinger cat [212] is a candidate for the role of another observer. Otherwise why did Schrödinger use such an eccentric example of a macroscopic device as a cat?... For a modern discussion see [257, 256, 94].

the quantum observer is of course idealized as well.[11] And this leads to some errors in describing the object and to a "gap" between a "working" quantum theory and its foundations and to the appearance of "accursed questions."

The classical observer is capable of directly perceiving the spacetime relations. This capability is simply postulated along with an unlimited resolving power, as well as an unlimited memory capacity and reliability. This leads to a classical picture of reality. In the quantum domain it is impossible to provide the observer with a high resolving power together with a good memory without the dynamics of the object being distorted beyond recognition. The Copenhagen interpretation attributes to the observer an ideal memory but a low resolving power (he sees only macroscopic bodies). The "objective" interpretation, on the contrary, imputes to the ideal observer an infinitely high resolving power but absolutely no memory for the storing of past observations. This, of course, is not true of real observers. They are treated as some complicated material systems to be described in the scope of the picture of reality given by an ideal observer. The relative-state interpretation was originally formulated[74] for an observer having ideal memory, permitting us to weaken this requirement[258, 154].

To describe a real observer in quantum theory, various alternatives and combinations of several interrelated concepts have been used: decomposition of the physical Universe into subsystems and of the Hilbert space into a tensor product[74, 244, 258, 266, 265], singling-out of collective degrees of freedom[244, 266, 265] or thermodynamic parameters[52]; propagation of the quantum correlations at a distance exceeding the characteristic sizes of an observer[244, 260, 15, 107]; "duplication" of macroscopic information[244, 266, 265]; and so on (it is difficult to single out independent components in these entangled concepts). When discussing merits and demerits of such ideas, it is important to keep in mind that an absolutely exact formulation is surely unattainable; there is an obstacle of fundamental, almost epistemological level. That is, on the one hand, the physical theory is in need of the concept of a generalized observer deprived of individual features, and on the other hand, idealizing the real observer in quantum theory inevitably misrepresents the dynamics of an observed object!

Until recently, the processing of information was believed to be necessarily dissipative. Now the possibility in principle of creating a nondissipative computer is admitted[76, 62, 194, 179]. Can such a computer be

[11] The Copenhagen interpretation prescribes the "reduction of the state vector." But at the same time it reserves the possibility of decreasing errors by means of shifting the quantum/classical boundary.

regarded as an observer?[12] If so, must its physical picture of reality be the same as ours? We scarcely know how we ourselves process information; nevertheless our sense organs, as well as our means of intercourse, are obviously dissipative—opaque to quantum coherence. In this regard we are dissipative observers. By fixing this point in the first principles of the theory, we shall discriminate against some observers. At the expense of this, the accuracy (or rather the fundamental boundary of attainable accuracy[13]) of our theory will be increased. "Macroscopic systems are inherently dissipative"—by these words from the paper of Caldeira and Leggett[36] is our approach to the problems of the quantum/classical correspondence determined.

Dissipation, considered on the quantum level, of course, is not a result of attempts to quantize the friction force point-blank (see [224]). Instead it is a manifestation of the delocalization of quantum correlations. The most obvious example is a photon radiated or reflected by a macroscopic body and flying away to infinity. It is, however, useful to keep in mind, for instance, that the friction between a solid black body and the thermal radiation filling up the surrounding space influences the quantum-mechanical motion of this body in roughly the same way as does the friction between this body and the thermal radiation locked up in its internal cavity (if any). In particular, in both cases the equations of the next section are applicable. The "propagation" of correlations and "locality" of observables must be understood in a thermodynamical rather than geometrical sense. Zurek[266] emphasizes this by the term "built-in environment." The Poincaré time for a finite system consisting of a macroscopically large number of particles far exceeds the duration of any feasible experiment. In this respect, it can be said that there exists an essentially irreversible propagation of correlations in a spatially bounded system. On the other hand, no quantum-mechanical system can be regarded as isolated from outer fields for such long times[265]; this returns us to the ideas of spatial infinity, continuous spectrum, and "true" irreversibility.

The problem of the foundations of stat-physical (and thereby thermodynamical) irreversible descriptions in the scope of quantum theory is discussed in Section 6. The thermal radiation locked in (or rather, nearly locked in) the internal cavity is a buffer capable of readily "absorbing"

[12]This question was discussed in [4, 195]. For such an observer the concept of reduction caused by the digitization of information by the observer, would be in order[117]. The continuous reduction presented in the next section is more natural for us.

[13]Many investigators regard some inaccuracy in setting an observer or his environment's state and hence the Universe as a whole to be basically necessary[196, 202]. An opposite point of view is characteristic of the objective interpretation.

the quantum correlations so as to then progressively "radiate" them away to infinity. The thermodynamical delocalization of correlations is always eventually related to a spatial one, but can occur much faster than the latter. On the other hand, the spatial delocalization of correlations can be partially reversible, and thereby become many times larger than the thermodynamical delocalization, as was pointed out by Joos[118] (see also [168, 169]). We can attribute to the reversible delocalization of correlations the elastic deformation of the environment induced by a gravitating body, the reversible polarization of the environment by an electrically charged body (see also [238, Sect. 6A]), the dressing of bare particles, etc. The criteria for the applicability of the classical description within quantum theory must rest only upon irreversible processes. Unfortunately, some works[260, 154, 116, 117] do not always give due consideration to separating the elastic and dissipative processes; as a consequence the results of some definite estimates, when uncritically interpreted, can lead to a grossly exaggerated picture of the rate of thermodynamical delocalization of correlations. There is now a solid foundation for calculating this rate.

The fluctuation-dissipation relations (FDR) were obtained in a pioneering Callen and Welton 1951 work [40] generalizing the Nyquist formula for voltage noise in electrical circuits. They are applicable to a wide class of physical systems near thermal equilibrium, such as the motion of a solid body in a liquid, that of an electrically charged particle in a thermal electromagnetic field, and others. Attention was chiefly given to the environment (liquid, field) described at the quantum level. The mechanical system served only as a "probe particle" to be described classically. Senitzky[218] put more emphasis on the mechanical system and gave a quantum description of its motion in the presence of a slight interaction with the environment. He has shown that for the approximation used, the environment can always be described by means of the canonical commutation relations (CCR), regarding its Hamiltonian as quadratic and the interaction Hamiltonian as linear in the environment variables.

Thus, mechanical systems with friction came within the reach of the quantum description, though a number of more delicate points were made more precise later (see, in particular, [155, 101, 207, 238]), and some of them are still poorly understood. It can be said that the environment affects the body via a sum of two forces. One, fluctuational, is random and has zero mean value and a definite covariance function; another, dissipative, is integrally dependent on the preceding body pathway, the kernel of the integral (Green function) being related to the covariance function of fluctuations through FDR. Both functions are localized on the time scale characteristic of microscopic processes in the environment. Usually the motion of the mechanical system is relatively slow. This allows us to return to a Langevin equation local in time. Agarval[3] found out that in such a situation the quantum dynamics is formally identical to the classical one, when written in the form of the Markov master equation for the Wigner distribution function. It is a partial differential equation of the Fokker-Planck type (see the next section). A closed derivation of the equations of quantum dynamics (non-Markovian to begin with, and then Markovian ones) for a system of collective degrees of freedom slightly interacting with thermal ones is contained[14] in our work [146]. Our treatment is for a rather general case, with an explicit analysis of initial assumptions but without seeking full mathematical rigor (see also [80, 81]). The environment-induced non-Hamiltonian effects, dissipation and fluctuation, exist against a background of (usually larger-magnitude) Hamiltonian effects (renormalization of the mass and other mechanical parameters). This is nothing but the above-discussed difference between the reversible and irreversible delocalization of correlations.

Collisions of heavy ions[108, 109, 261] have become a remarkable testing ground for this theory. The objects here are small enough to require the quantum description, and at the same time large enough to entail dissipation and fluctuations. In [108] the description which is local in time is given by a system of differential equations the order of which is dependent on properties of the environment. It is noted[108, 109, 261] that usually the environment also possesses low-frequency collective modes which cause an essential delay. Some effects are discussed which are usually negligible in macroscopic situations: the influence of mechanical motion on the environment temperature, a nonadiabatic reaction of the environment on the mechanical motion, and so on. For FDR in mesoscopic domain see [85].

A new wave of interest in the quantum description of dissipative processes had already arisen in the early 80's in connection with the problems of the quantum/classical correspondence[168, 36, 167, 174, 119, 263, 146] and [37, 38, 245, 211, 68, 65, 184]. There is some hope of realizing experimentally the quantum behavior of a macroscopic degree of freedom, e.g., the magnetic flux in a SQUID or a phase order parameter in a Josephson junction[149, 5]; the corresponding experiments will be discussed in Section 7. Such systems are of value not only because of their small dissipation, but also because of the availability of microscopically small details of the potential pattern, that is, wells and barriers. The quantum motion

[14] Unfortunately, in [146] there is some inconsistency in the notation. The following corrections should be made: a factor of $h/2$ before the function g, and before the three indexed value $\mathcal{C}$ in Section 4.7 and 4.8. In addition. the third integral in (8) is taken over $t_1 > t_2$ rather than $t_2 > t_1$. We regret the inconvenience.

in such a potential can be quite different from the above-mentioned quasi-classical motion with dissipation and fluctuations, because the former includes tunnel transitions. It is, however, also different from that described by the WKB method, for the latter does not allow us to take into account dissipative effects. New methods, predicting new phenomena, have been developed[214, 44, 96, 167]. We will not dwell on these interesting questions, as we are not sure one should call "macroscopic" a degree of freedom whose potential pattern has microscopically minute details essentially affecting the motion. More important for our purposes is to conclude that the description of the environment using CCR (in other words, with the use of harmonic oscillators) is widely applicable[215, 36, 97]. In nonlinear optics the possibility of creating and detecting superpositions of macroscopically different states has recently been recognized[254, 232, 121].

The lack of manifestations of quantum coherence in the macro-world can thus be understood as the result of the delocalization of quantum correlations due to interactions with the environment, and this interaction can usually be calculated with FDR. Does this mean that a certain dimensionless "quantalness coefficient" for a given macroscopic process can be calculated? What would serve as such a coefficient? In 1979 Wotters and Zurek[253] quantitatively analyzed the double-slit interference experiment where, going back to Einstein, a particle and its environment interacted rather weakly so that the fringe pattern was not too spoiled. They have shown that such an interaction can be sufficient for an indirect measurement of the coordinate, giving incomplete but considerable information on which slit the particle has gone through. The influence of the environment may be estimated by the destructive effect which it has on one or another manifestation of quantum coherence. While the interference pattern was chosen in [253], in our 1985 work[147] the violation of Bell's inequalities was chosen for that purpose. Assuming that the environment introduces into the coordinate and momentum of a particle fluctuations which are independent, normally distributed random variables with standard deviations $\Delta q, \Delta p$, we have shown that the highest possible value of a violation of Bell's inequalities is small when $\Delta p \Delta q \gg h$. In this case Bell's inequalities are considered for quite arbitrary two-valued observables. The interaction with the environment is assumed linear in the coordinates and momenta; however, later, when the measurements are made, the linear functions in the coordinates and momenta are not singled-out among the nonlinear ones. The expression "the highest possible violation of Bell's inequalities" implies an optimization (finding a maximum) both in initial states and measured observables. This requires more general mathematical methods than those necessary to calculate a particular effect, e.g., an interference pattern.

Dissipation has taken its place in investigations since 1985[38, 119, 245, 211, 263]. Caldeira and Leggett[38] have considered the harmonic oscillator whose initial state is assumed to be the superposition of two strongly different coherent states, ground (fixed) and shifted (oscillating). Each time the oscillating wave packet comes close to a zero point, the probability distribution for the coordinate develops fringes due to interference. The destructive influence of a dissipative environment on this interference is studied quantitatively with the use of FDR, as a function of six parameters: the environment (reservoir) temperature T, the friction coefficient Γ, the oscillator frequency, its mass m, the initial shift of the oscillating wave packet, and the upper frequency of the reservoir. The presence of four dimensionless parameters changing within wide limits makes investigation [38] rich in content, but its results difficult to perceive and utilize. In the works of Joos and Zeh[119], Walls and Milburn[245], Savage and Walls[211], and Zurek[263] the number of parameters is reduced to four (two are dimensionless) at the cost of the substitution of an oscillator by a free particle and the utilization of a Markov approximation adequate at not-so-low temperatures. The delocalization rate of quantum correlations is characterized by the quantity $\Lambda = h^{-2}k_B T\Gamma$, of dimensions $[l^{-2}t^{-1}]$ (for details, see the end of next section). The parts of a wave packet which are spaced Δx apart lose their coherence after a time of the order $\Lambda^{-1}(\Delta x)^{-2}$. This is some reply to the question used by Zurek as a title of [263], "Reduction of the wavepacket: how long does it take?" In our 1987 work[146] the same question was answered differently, with no Δx parameter: the coherence vanishes within a time of order $\sqrt{hm/(k_B T\Gamma)}$. The difference in the results is accounted for by different ways of formulating the problem. The absence of extra parameters like Δx in our approach, which is based on Bell's inequalities, is concerned with the already-mentioned optimization in the observables and states. The time given by us serves not only coordinate observables but momentum ones as well, or any functions of coordinates and momenta.

No matter how the "quantalness coefficient" is defined, it is natural to expect it to tend to zero when the fluctuational influence of the environment tends to infinity. Quasi-classical methods have trained us to think that the quantum corrections to the classical predictions happen to be small but not zero. It is from this position that we discussed in [147] the quantum corrections to Bell's inequalities. Here, a welcome surprise awaited us! These corrections proved[146] to vanish exactly, when $\Delta q \Delta p \geq h$. This means that the characteristic time of destroying the coherence can be understood not only as the time for the essential weakening of the quantum effects but rather as that of their *utter vanishing.* At this length of time the environment engenders a change in the quantum state that is equivalent to the absorption and reemission of the given quantum system by some classical apparatus. In other words, it is equivalent to a sequence "measuring/preparing," which transforms quantum states into quantum ones, as contrasted with the usual sequence "preparing/measuring," which transforms classical states into classical ones. An exact definition of this notion of "classical factorization" was given in [146] and will be discussed in the next section. Such a process completely destroys any quantum correlations of the given system with other systems (of course, not including in itself the environment having caused the factorization); this leads to Bell's inequalities being satisfied.

Thus, in some situations the quantum corrections utterly vanish. Does this mean that the system in such situations can be described classically in the sense explained in Section 2 (admitting a classical superstructure over quantum dynamics)? Yes, there is such a possibility, as was indicated in 1987 by Diosi[68] and by us[146]. Our approach[146] is as follows: the evolution of the system during a time longer than that of classical factorization can be presented as a sequence of alternating acts of measuring and preparing. The measurement outcomes form a "dotted-in-time" classical description. There is no classical determinacy, but also no quantum interference. In [146] we indicated a way to estimate the time of classical factorization of various particular models. As a rule, it is close to the above-mentioned value $\sqrt{hm/k_B T \Gamma}$ (see Sections 4 and 7). As for the approach of Diosi[68, 65] see the next section, which shows that its unification with the approach of our work[146] is not only possible but fruitful.

4 Dissipative Dynamics of Correlations

The previous section pointed to the necessity of considering the quantum dynamics of a macroscopic system by taking into account the fluctuation forces acting on it from the environment and obeying the fluctuation-dissipation relations. Let us stress that the classically looking fluctuation force results from a *quantum* interaction with a *quantum* environment (see Section 3). Below we shall consider how the influence of the fluctuation forces is manifested in the investigation of the problem of reconstructing the classical world within the quantum description, using a method which develops the approach of Diosi[68, 65] and ours [146]. (See also works of Barchielli, Lanz, Prosperi, and Lupieri cited in [65, 35, 203], and Section 3 of Ghirardi, Rimini, and Weber's work[88] and [245, 211].)

Let us begin with the simplest case. Let there be one collective degree of freedom, namely, the coordinate q and the momentum p of a body of mass m. The body is affected by the environment through a fluctuation force F_{fluc} proportional to a white noise,

$$\langle F_{\text{fluc}}(s) F_{\text{fluc}}(t) \rangle = \lambda^2 \delta(s-t) \tag{4.1}$$

The fluctuation-noise intensity, dependent on the environment temperature and the friction coefficient (see the end of this section), can be described by the constant λ or by the following value having the dimensions of time:

$$\tau_{\text{fluc}} = \frac{\sqrt{hm}}{\lambda} \tag{4.2}$$

Other forces are assumed to be absent. The classical equations of motion, $\dot{p} = F_{\text{fluc}}$, $\dot{q} = p/m$ can be written in the form of stochastic differential equations (about this notion see, for example, [176])

$$\begin{aligned} dp &= \lambda \, db \\ dq &= (1/m) p \, dt \end{aligned} \tag{4.3}$$

where $b(t)$ is the Wiener random process whose derivative is the white noise: $\langle \dot{b}(s) \dot{b}(t) \rangle = \delta(s-t)$. The proper evolution equation for the classical probability density $w(p,q,t)$ in phase space is well known:

$$\left(\frac{\partial}{\partial t} - \frac{\lambda^2}{2} \frac{\partial^2}{\partial p^2} + \frac{1}{m} p \frac{\partial}{\partial q} \right) w(p,q,t) = 0 \tag{4.4}$$

The same equation describes the dynamics of the quantum density matrix if $w(p,q,t)$ denotes a Wigner distribution function (in other words, a Weyl symbol of the density matrix); see, e.g., [24]. Thus, Eq. (4.4) determines the quantum dynamics of a macroscopic body in the presence of fluctuation forces.

Let the distribution w be Gaussian (i.e., the exponent of a second-degree polynomial of in p and q) at some moment; it will then remain Gaussian at all later moments. Pure Gaussian quantum states are also known as *coherent states*, and mixed ones as *Gibbs states* for the quadratic Hamiltonians. The Gaussian distribution (and hence the Gaussian quantum state) is unambiguously determined by the first and second moments $\langle p \rangle$, $\langle q \rangle$, $\langle p^2 \rangle$, $\langle pq \rangle$, $\langle q^2 \rangle$. The central second moments $\langle p_0^2 \rangle, \langle p_0 q_0 \rangle, \langle q_0^2 \rangle$ are more convenient to consider than the original second moments $\langle p^2 \rangle, \langle pq \rangle, \langle q^2 \rangle$. Here and below,

$$\begin{aligned} p_0 &\equiv p - \langle p \rangle \\ q_0 &\equiv q - \langle q \rangle \end{aligned}$$

The dynamics of the moments, corresponding to Eq. (4.4), is more conveniently calculated from (4.3)—but keeping in mind the rules of the Ito stochastic differential calculus: $(db)^2 = dt$, $dbdt = 0$, $(dt)^2 = 0$, and also $\langle f db\rangle = 0$ for any nonanticipating functional f of b (see, e.g., [176].) We obtain

$$\begin{aligned}
\frac{d}{dt}\langle p\rangle &= 0 \\
\frac{d}{dt}\langle q\rangle &= \frac{1}{m}\langle p\rangle \\
\frac{d}{dt}\langle p_0^2\rangle &= \lambda^2 \\
\frac{d}{dt}\langle p_0 q_0\rangle &= \frac{1}{m}\langle p_0^2\rangle \\
\frac{d}{dt}\langle q_0^2\rangle &= \frac{2}{m}\langle p_0 q_0\rangle
\end{aligned} \tag{4.5}$$

All quantum states obey a known inequality for the discriminant

$$\langle p_0^2\rangle\langle q_0^2\rangle - \langle p_0 q_0\rangle^2 \geq \frac{h^2}{4} \tag{4.6}$$

This is, of course, one of the forms of the uncertainty relation (see, e.g., [110]). Equality is attained for pure Gaussian states. The dynamics (4.4) transforms pure states into mixed ones and enlarges the discriminant. Thus, it follows from (4.5) that

$$\frac{d}{dt}\left(\langle p_0^2\rangle\langle q_0^2\rangle - \langle p_0 q_0\rangle^2\right) = \lambda^2\langle q_0^2\rangle$$

It is convenient to represent each Gaussian state by an ellipse on a phase plane— known in probability theory as a *concentration ellipse*. The point (p_1, q_1) belongs to this ellipse if $|ap_1 + bq_1 + c| \leq 1$ for any a, b, c such that $\langle ap+bq+c\rangle = 0$ and $\langle (ap+bq+c)^2\rangle \leq 1$. The area of the concentration ellipse is proportional to the square root of the discriminant. For a pure state, it is equal to $\pi h/2$. A mixed Gaussian state can be presented as a mixture of pure Gaussian ones by the formula

$$w_1(p, q) = \int w_0(p - p', q - q')\gamma(p', q')dp'dq' \tag{4.7}$$

where γ is an appropriate Gaussian distribution, $\langle p\rangle_1 = \langle p\rangle_0 + \langle p\rangle_\gamma$, $\langle p_0^2\rangle_1 = \langle p_0^2\rangle_0 + \langle p_0^2\rangle_\gamma$, and similar relations hold for q, $p_0 q_0$, and q_0^2. Expansion (4.7) with the given w_0, w_1 exists if the concentration ellipse for w_0 lies inside that for w_1. It is clear that for a given w_1 there is a great arbitrariness in the choice of w_0.

We shall realize an idea suggested in the 1987 work of Diosi[68] which, using the notation introduced above, can be formulated as follows. Take as w_1 the mixed Gaussian state obtained from the given pure Gaussian one w_0 by the use of evolution (4.4) for an infinitesimal time dt. Now decomposition (4.7) shows that the evolution of a quantum state is reduced[15] to its shift on the phase plane by a Gaussian random vector (p', q') with the parameters

$$\begin{aligned}
\langle p'\rangle_\gamma &= d\langle p\rangle \\
\langle q'\rangle_\gamma &= d\langle q\rangle \\
\langle p_0'^2\rangle_\gamma &= d\langle p_0^2\rangle \\
\langle p_0' q_0'\rangle_\gamma &= d\langle p_0 q_0\rangle \\
\langle q_0'^2\rangle_\gamma &= d\langle q_0^2\rangle
\end{aligned} \tag{4.8}$$

which according to (4.5) are equal to

$$\begin{aligned}
\langle p'\rangle_\gamma &= 0 \\
\langle q'\rangle_\gamma &= \frac{1}{m}\langle p\rangle dt \\
\langle p_0'^2\rangle_\gamma &= \lambda^2 dt \\
\langle p_0' q_0'\rangle_\gamma &= \frac{1}{m}\langle p_0^2\rangle dt \\
\langle q_0'^2\rangle_\gamma &= \frac{2}{m}\langle p_0 q_0\rangle dt
\end{aligned} \tag{4.9}$$

Integrating these differentials we obtain for the evolution of the quantum state a description which will be called a *diffusional-coherent* one:

$$w(p, q, t) = \mathbf{E}\, w_0(p - p_{\mathrm{cl}}(t), q - q_{\mathrm{cl}}(t)) \tag{4.10}$$

[15] Of course, a mixed state can be decomposed into a mixture of pure states in various ways. We shall limit ourselves to the Gaussian states. For other possibilities, see [66].

Here $\mathbf{E}$ denotes averaging over the classical random process $p_{\text{cl}}(t), q_{\text{cl}}(t)$ on the phase plane— a diffusion process with the parameters obtained as follows from (4.9):

$$\begin{aligned}
\mathbf{E}\, dp_{\text{cl}} &= 0 \\
\mathbf{E}\, dq_{\text{cl}} &= \frac{1}{m} p_{\text{cl}}\, dt \\
(dp_{\text{cl}})^2 &= \lambda^2\, dt \\
(dp_{\text{cl}})(dq_{\text{cl}}) &= \frac{1}{m} \langle p_0^2 \rangle\, dt \\
(dq_{\text{cl}})^2 &= \frac{2}{m} \langle p_0 q_0 \rangle\, dt
\end{aligned} \tag{4.11}$$

where the averages on the right-hand sides are taken over the pure Gaussian state w_0 used in (4.10).

Not all pure Gaussian states can be used for the diffusional-coherent description, but only those for which, for all a and b,

$$\left.\frac{d}{dt}\right|_{t=0} \langle (ap_0 + bq_0)^2 \rangle \geq 0. \tag{4.12}$$

In other words, a change of the distribution w_0 (or its concentration ellipse) following (4.4) must be an expansion in all directions. Condition (4.12) is fulfilled if, and only if,

$$\begin{aligned}
&\left.\frac{d}{dt}\right|_{t=0} \langle p_0^2 \rangle \geq 0 \\
&\left.\frac{d}{dt}\right|_{t=0} \langle q_0^2 \rangle \geq 0 \\
&\left(\left.\frac{d}{dt}\right|_{t=0} \langle p_0 q_0 \rangle \right)^2 \leq \left(\left.\frac{d}{dt}\right|_{t=0} \langle p_0^2 \rangle \right) \left(\left.\frac{d}{dt}\right|_{t=0} \langle q_0^2 \rangle \right)
\end{aligned} \tag{4.13}$$

Using (4.5), we obtain the condition

$$\langle p_0^2 \rangle \leq 2m\lambda^2 \langle p_0 q_0 \rangle \tag{4.14}$$

What does the dynamics of correlations have to do with the above? The answer becomes clear as soon as the diffusional-coherent description is applied to a system possessing several degrees of freedom. The point is that a state of the form

$$\begin{aligned}
w(p_1, q_1, p_2, q_2, t) = \mathbf{E}\, & w_0^{(1)}(p_1 - p_1^{\text{cl}}(t), q_1 - q_1^{\text{cl}}(t)) \\
& \times w_0^{(2)}(p_2 - p_2^{\text{cl}}(t), q_2 - q_2^{\text{cl}}(t))
\end{aligned} \tag{4.15}$$

belongs to a class of *decomposable* states (the term is taken from [11]), i.e., to that of factorized states and their mixtures. But for such quantum states, as mentioned in Section 1, the classical Bell inequalities are valid. Formula (4.15) is obtained by an obvious generalization of the diffusional-coherent description to systems with several degrees of freedom, provided that a supplementary condition is fulfilled, namely that the coherent state of all the system w_0 is factorized, that is, equal to the tensor product of two states, $w_0^{(1)}$ and $w_0^{(2)}$, each related to one degree of freedom. Any diffusional-coherent description satisfying this condition (whose feasibility will be analyzed below) will be called a *diffusional-coherent decomposition.*

We emphasize that state (4.15) is not at all factorizable. There are correlations between the two subsystems, but classical rather than quantum ones, in the sense that they can be reduced to the correlations between classical random processes $(p_1^{\mathrm{cl}}(t), q_1^{\mathrm{cl}}(t))$ and $(p_2^{\mathrm{cl}}(t), q_2^{\mathrm{cl}}(t))$. For macroscopic low-dissipation systems, this correlation is in many cases so strong that it becomes close (at the classical level) to a functional dependence between $(p_1^{\mathrm{cl}}(t), q_1^{\mathrm{cl}}(t))$ and $(p_2^{\mathrm{cl}}(t), q_2^{\mathrm{cl}}(t))$. But, however strong, the classical correlation cannot lead to any violation of Bell's inequalities even by the slightest amount. This explains our interest in diffusional-coherent decompositions, an interest that is not diminished by an arbitrariness existing in the choice of decomposition parameters; because in the Bell inequalities the arbitrariness no longer exists.

The conclusion on missing quantum correlations is of interest when there is an interaction. Consider a Hamiltonian of the form

$$\begin{aligned} H(p_1, q_1, p_2, q_2) = \frac{1}{2m_1}p_1^2 + \frac{1}{2m_2}p_2^2 \\ +\frac{1}{2}k_1 q_1^2 + \frac{1}{2}k_2 q_2^2 + k_{12} q_1 q_2 \end{aligned} \tag{4.16}$$

Adding the fluctuations and assuming for simplicity that the fluctuation forces which affect different degrees of freedom do not correlate with each other, we obtain instead of (4.3) the following equations of motion:

$$\begin{aligned} dp_1 &= -k_1 q_1 dt - k_{12} q_2\, dt + \lambda_1\, db_1 \\ dq_1 &= (1/m_1) p_1\, dt \\ dp_2 &= -k_2 q_2\, dt - k_{12} q_1\, dt + \lambda_2\, db_2 \\ dq_2 &= (1/m_2) p_2\, dt \end{aligned} \tag{4.17}$$

Calculating $d/dt\,|_{t=0}\, \langle (a_1 p_1 + b_1 q_1 + a_2 p_2 + b_2 q_2)^2 \rangle$ and holding, for simplicity, the first-order moments to be equal to zero, we obtain a condition of fitness of a factorized state for the diffusional-coherent decomposition;

namely, for all a_1, b_1, a_2, b_2,

$$\begin{aligned}
&(\lambda_1^2 - 2k_1\langle p_1q_1\rangle)a_1^2 + (\lambda_2^2 - 2k_2\langle p_2q_2\rangle)a_2^2 \\
&\quad + 2\left(\frac{1}{m_1}\langle p_1^2\rangle - k_1\langle q_1^2\rangle\right)a_1b_1 + 2\left(\frac{1}{m_2}\langle p_2^2\rangle - k_2\langle q_2^2\rangle\right)a_2b_2 \\
&\quad + \frac{2}{m_1}\langle p_1q_1\rangle b_1^2 + \frac{2}{m_2}\langle p_2q_2\rangle b_2^2 \\
&\quad - 2k_{12}(\langle p_1q_1\rangle + \langle p_2q_2\rangle)a_1a_2 - 2k_{12}\langle q_1^2\rangle b_1a_2 \\
&\quad - 2k_{12}\langle q_2^2\rangle a_1b_2 \geq 0
\end{aligned} \tag{4.18}$$

Simultaneously, the "uncertainty relations"

$$\begin{aligned}
\langle p_1^2\rangle\langle q_1^2\rangle - \langle p_1q_1\rangle^2 &= \frac{h^2}{4} \\
\langle p_2^2\rangle\langle q_2^2\rangle - \langle p_2q_2\rangle^2 &= \frac{h^2}{4}
\end{aligned} \tag{4.19}$$

must be fulfilled. It would be surprising if the nonnegativity of the four-variable quadratic form (4.18) were as simply expressed as in (4.12)–(4.14). Nevertheless, the issue of the existence of the diffusional-coherent decomposition is finally settled by the following

Theorem 1. The inequality

$$|k_{12}| \leq \frac{1}{h}\lambda_1\lambda_2 \tag{4.20}$$

is a necessary and sufficient condition imposed on m_1, m_2, λ_1, λ_2, k_1, k_2, k_{12} for the existence of moments $\langle p_1^2\rangle$, $\langle p_1q_1\rangle$, $\langle q_1^2\rangle$, $\langle p_2^2\rangle$, $\langle p_2q_2\rangle$, $\langle q_2^2\rangle$ that satisfy equalities (4.19) and ensure a nonnegative form (4.18).

The proof will be published [234]. In the "extreme" case where (4.20) becomes an equality, the unknown moments obey the equations

$$\begin{aligned}
\frac{\langle p_nq_n\rangle}{\langle q_n^2\rangle^2} &= \frac{m_n\lambda_n^2}{h^2} \\
\frac{h}{2\langle q_np_n\rangle} - \frac{2\langle q_np_n\rangle}{h} &= \frac{2hk_n}{\lambda_n^2}
\end{aligned} \tag{4.21}$$

which, in conjunction with (4.19), unambigously determine these moments. The state determined by equalities (4.19) and (4.21) ensures the diffusional-coherent decomposition over all our range (4.20) for k_{12}. Outside this range condition (4.18) cannot be satisfied for factorized quantum states, whether pure or mixed. It is noteworthy that (4.20) does not contain k_1, k_2.

Now, the dynamics (4.16)–(4.17) permits a decomposition of the form (4.15), provided that condition (4.20) is fulfilled. This follows from the above, provided, however, that the initial state of the system can be presented as a mixture of proper-form coherent states. And what if the initial state is arbitrary? The diffusional-coherent decomposition is applicable in this case as well, but from a definite later moment on, rather than from the initial moment. The fact is that the dynamics considered transforms an arbitrary quantum state to a mixture of coherent states in a finite time. We will discuss this in more detail.

The transformation of a quantum state at finite time t according to the dynamics (4.4) may be decomposed into the product of two transformations: the first is the unitary operator which corresponds to the linear canonical transformation (deduced from (4.4) if λ is substituted by zero); the second, in the language of Wigner distributions, has the form of convolution [as in (4.7)] with a definite Gaussian measure γ_t, whose parameters satisfy Eqs. (4.5) with zero initial ($t = 0$) conditions. The first (unitary) transformation transforms the arbitrary initial state into another one which is also arbitrary. The important thing is that the second transformation (which, obviously, transforms pure states into mixed ones) takes place. Its properties depend on the parameters of the measure γ_t. All of this remains valid in a more general situation such as (4.17), provided of course, that Eqs. (4.5) are appropriately generalized. The measure γ_t coincides with the probability distribution for the state of the classical counterpart of the given quantum system.

Consider the case of one degree of freedom in the presence of a quadratic potential:

$$\begin{aligned} dp &= -kq \cdot dt + \lambda \cdot db \\ dq &= \frac{1}{m} p \cdot dt \end{aligned} \tag{4.22}$$

Solving the corresponding equations for moments

$$\begin{aligned} (d/dt)\langle p_0^2 \rangle &= -2k\langle p_0 q_0 \rangle + \lambda^2 \\ (d/dt)\langle p_0 q_0 \rangle &= (1/m)\langle p_0^2 \rangle - k\langle q_0^2 \rangle \\ (d/dt)\langle q_0^2 \rangle &= (2/m)\langle p_0 q_0 \rangle \end{aligned} \tag{4.23}$$

under zero initial conditions, we find the parameters of the measure γ_t.

For $k > 0$, we obtain

$$\begin{aligned}
\langle p_0^2 \rangle &= \frac{\lambda^2 t}{2}\left(1 + \frac{\tau_{\text{mech}}}{2t}\sin\frac{2t}{\tau_{\text{mech}}}\right) \\
\langle p_0 q_0 \rangle &= \frac{\lambda^2 t^2}{2m}\left(\frac{\tau_{\text{mech}}}{t}\sin\frac{t}{\tau_{\text{mech}}}\right)^2 \\
\langle q_0^2 \rangle &= \frac{\lambda^2 t}{2m^2}\tau_{\text{mech}}^2\left(1 - \frac{\tau_{\text{mech}}}{2t}\sin\frac{2t}{\tau_{\text{mech}}}\right)
\end{aligned} \tag{4.24}$$

where $\tau_{\text{mech}} = \sqrt{m/k}$. When $k < 0$, the formulas are similar, with the sines being hyperbolic. This latter case, known as an "upside down" oscillator, has recently attracted a great deal of attention [98, 91, 100, 238, 262]. Understood literally, it corresponds to an energy spectrum unbounded from below. However, it is useful for the approximation near the potential-energy maximum. Indeed, the approximating Hamiltonian $H(p,q) = (1/2m)p^2 + \frac{1}{2}kq^2$ with $k < 0$ differs from a true Hamiltonian $H_{\text{true}}(p,q) = (1/2m)p^2 + U(q)$, $\inf U(q) > -\infty$, by an unbounded difference in potentials $U(q) - \frac{1}{2}kq^2$. Doubts may be raised that the approximation is adequate. So let us show how the error can be estimated for the non-stationary problem when the approximating wave packet remains mainly in the region where $U(q) \approx \frac{1}{2}kq^2$. The estimations below hold for any $k \in (-\infty, +\infty)$. Let $ih(d/dt)\psi_{\text{true}}(t) = H_{\text{true}}(t)\psi_{\text{true}}(t)$, $ih(d/dt)\psi(t) = H(t)\psi(t)$ (time-dependent Hamiltonians include the potential of the fluctuation force); then $ih(d/dt)(\psi_{\text{true}}(t) - \psi(t)) = H_{\text{true}}(t)(\psi_{\text{true}}(t) - \psi(t)) + (H_{\text{true}}(t) - H(t))\psi(t)$. The inequality for norm follows:

$$h\frac{d}{dt}|\psi_{\text{true}}(t) - \psi(t)| \leq |(H_{\text{true}}(t) - H(t))\psi(t)|$$

The operator $H_{\text{true}}(t) - H(t)$ is a function of the coordinate operator, the function $U(q) - \frac{1}{2}kq^2$ being $\mathrm{O}(q^2)$ for large q and $\mathrm{o}(q^2)$ for small q. The distribution of the observable q for the state $\psi(t)$ coincides with the distribution of a linear combination of p, q for the initial state $\psi_0 = \psi(0) = \psi_{\text{true}}(0)$ due to the quadratic nature of $H(t)$. Now it is easy to find conditions for ψ_0 (in terms of distributions of p, q) ensuring that $|\psi_{\text{true}}(t) - \psi(t)| \ll 1$ for moderate t [namely for $t \leq \tau_f$; see (4.29) below]. We restrict ourselves again to quadratic Hamiltonians.

Theorem 2. If the Gaussian measure γ satisfies the inequality for the discriminant

$$\langle p_0^2 \rangle_\gamma \langle q_0^2 \rangle_\gamma - \langle p_0 q_0 \rangle_\gamma^2 \geq \frac{h^2}{4} \tag{4.25}$$

then the transformation of the convolution with the measure γ transforms any Wigner distribution function into a non-negative function.

The proof given by us in [146] shows that this theorem is essentially a reformulation of the known relation between Weyl and Wick symbols (see, e.g., [24]).

Theorem 3. Let w_0 be a Wigner distribution function of a pure Gaussian state and let γ be a Gaussian measure satisfying the inequalities

$$\begin{gathered}\langle p_0^2\rangle_\gamma \ge \langle p_0^2\rangle_{w_0}\\ \langle q_0^2\rangle_\gamma \ge \langle q_0^2\rangle_{w_0} \qquad (4.26)\\ (\langle p_0^2\rangle_\gamma - \langle p_0^2\rangle_{w_0})(\langle q_0^2\rangle_\gamma - \langle q_0^2\rangle_{w_0}) - (\langle p_0 q_0\rangle_\gamma - \langle p_0 q_0\rangle_{w_0})^2 \ge h^2/4\end{gathered}$$

Then the transformation of convolution with the measure γ transforms any Wigner distribution function w into a function of the form

$$w_1(p,q) = \int w_0(p-p', q-q') \cdot f(p',q')dp'dq' \qquad (4.27)$$

with a non-negative function f.

Proof. Represent γ as a convolution of w_0 and Gaussian measure γ_0; take for f the convolution of w with γ_0; note that non-negativity of this function is assured by Theorem 2.

If we do not want to fix *a priori* w_0, but seek a representation in the form (4.27) with some w_0 and f, then the condition

$$\langle p_0^2\rangle_\gamma \langle q_0^2\rangle_\gamma - \langle p_0 q_0\rangle_\gamma^2 \ge h^2 \qquad (4.28)$$

can be utilized instead of (4.26). The second moments for γ are twice as large as those for w_0, i.e., $\gamma_0 = w_0$. This is a particular case (for one degree of freedom) of Theorem 3.4 from [146].

The transformation of a quantum state which is expressed in Wigner distribution language as a convolution with a Gaussian measure satisfying condition (4.28) belongs to the class of classically factorizable operations, which we have introduced in [146], and which was discussed in the previous section. Such a transformation transforms any quantum state into a decomposable one, and this implements the classical Bell inequalities. The time during which dissipative dynamics leads to such a result is that of the *classical factorization* τ_f. See [146] for some precise definitions and theorems.

Substituting (4.24) into (4.28), one can estimate the order of magnitude of τ_f for the dynamics (4.22):

$$\begin{aligned} &\tau_f \sim \tau_{\text{fluc}} \quad \text{when } \tau_{\text{fluc}} \ll \tau_{\text{mech}} \\ &\tau_f \sim \frac{\tau_{\text{fluc}}^2}{\tau_{\text{mech}}} \quad \text{when } \tau_{\text{fluc}} \gg \tau_{\text{mech}} \text{ and } k > 0 \\ &\tau_f \sim \tau_{\text{mech}} \log \frac{\tau_{\text{fluc}}}{\tau_{\text{mech}}} \quad \text{when } \tau_{\text{fluc}} \gg \tau_{\text{mech}} \text{ and } k < 0 \end{aligned} \tag{4.29}$$

However, it is desirable to obtain the mixture of such states as are determined by relations (4.21), (4.19), rather than of some coherent states. We limit ourselves to a “symmetrical” case:

$$m_1 = m_2, \quad \lambda_1 = \lambda_2, \quad k_1 = k_2$$

Then the normal modes

$$\begin{aligned} q_\pm &= (q_1 \pm q_2)/\sqrt{2} \\ p_\pm &= (p_1 \pm p_2)/\sqrt{2} \end{aligned}$$

evolve independently of each other, and the dynamics of each mode has the form (4.22) with the same m and λ as for the initial degrees of freedom but with a different k, that is, $k_\pm = k \mp k_{12}$. It remains to substitute (4.24) into (4.26), defining w_0 through (4.21). At first sight, one has no doubts that when t is large enough the required inequalities are satisfied independently of the properties of w_0, because the right-hand sides of (4.24) increase without bound as $t \to \infty$. There is, however, an “obstacle.” Consider the concentration ellipse corresponding to (4.24). As $t \to \infty$, this ellipse expands without bound in all directions (though nonuniformly), and therefore sooner or later covers the concentration ellipse for w_0, whatever the properties of the latter. However, the limit of the concentration ellipse at $k < 0$ is a band of finite width[238, 262] rather than the entire plane. Indeed, from the hyperbolic analog of (4.24) there follows the equality

$$\langle ((-km)^{-1/4} p_0 - (-km)^{1/4} q_0)^2 \rangle = \frac{\lambda^2}{-2k} \left(1 - \exp\left(\frac{-2t}{\tau_{\text{mech}}} \right) \right) \tag{4.30}$$

In this case the criterion for fulfilling inequalities (4.26) at large t has the form

$$\langle ((-km)^{-1/4} p_0 - (-km)^{1/4} q_0)^2 \rangle_{w_0} < -\frac{\lambda^2}{2k} \tag{4.31}$$

Theorem 4. Let $k < 0$, $|k_{12}| \leq \lambda^2/h$, and let the pure Gaussian state w_0 have the second central moments determined by the equalities

$$\frac{\langle p_0 q_0 \rangle}{\langle q_0^2 \rangle^2} = \frac{m\lambda^2}{h^2}$$
$$\frac{h}{2\langle p_0 q_0 \rangle} - \frac{2\langle p_0 q_0 \rangle}{h} = \frac{2h(k + k_{12})}{\lambda^2}$$

Then the state w_0 satisfies inequality (4.31).

The proof will be published elsewhere. Theorem 4 guarantees that in the "symmetrical" two degree of freedom case the arbitrary initial state goes at finite time into a coherent-state mixture which permits applying the diffusional-coherent decomposition. It can be shown that the required time again has the order of magnitude expressed in (4.29). Note that, unlike (4.20), expression (4.29) essentially depends on k (through τ_{mech}).

To conclude this section, consider the fluctuation-dissipation relation relating the intensity of fluctuation-noise λ to the friction coefficient Γ and the environment temperature T,

$$\lambda^2 = 2\Gamma k_B T \tag{4.32}$$

where k_B is Boltzmann's constant. The criterion

$$\frac{|k_{12}|}{\Gamma T} \leq 2\frac{k_B}{h} \tag{4.33}$$

results from (4.20) and (4.32) when $\Gamma_1 = \Gamma_2$ and $T_1 = T_2$. This criterion was announced in the Introduction (with k_{12} denoting by A and assumed positive). It may also be written as

$$\tau_{\text{fluc}} \leq \tau_{\text{mech}} \tag{4.34}$$

with $\tau_{\text{fluc}} = \sqrt{hm}/\lambda$ [see (4.2)] and $\tau_{\text{mech}} = \sqrt{m/|k_{12}|}$. Note that, using four quantities $h, m, \Gamma, k_B T$, one can form one dimensionless combination

$$\varepsilon = \frac{h\Gamma}{m k_B T}$$

and any three of these four values allow us to form one and only one combination of any desired dimension. Good examples are the thermal de Broglie wavelength

$$\lambda_{\text{therm}} = h/\sqrt{2\pi m k_B T}$$

used in [119, 263], the wave-packet length

$$(\frac{h^3}{2m\lambda^2})^{1/4} = \sqrt{\pi}\lambda_{\text{therm}}\varepsilon^{-1/4}$$

obtained with (4.21) at $k = 0$, the relaxation time

$$\tau_{\text{fric}} = \frac{m}{\Gamma}$$

the characteristic period of thermal oscillations

$$\tau_{\text{therm}} = \frac{h}{k_B T}$$

and the time τ_{fluc} introduced by (4.2). Due to relation (4.32), we obtain

$$2\tau_{\text{fluc}}^2 = \tau_{\text{therm}} \cdot \tau_{\text{fric}}$$

(see formula (21) from [146]). Other examples are the classical diffusion coefficient $\mathcal{D}_{\text{class}} = k_B T/\Gamma$ (Einstein equation) and the quantum diffusion coefficient $\mathcal{D}_{\text{quant}} = h/m$ deduced from (4.11) when $\langle p_0 q_0 \rangle = h/2$. The dimensionless combination $\varepsilon = \tau_{\text{therm}}/\tau_{\text{fric}} = \mathcal{D}_{\text{quant}}/\ \mathcal{D}_{\text{class}}$ is small in the thermal (high-temperature) case and large in the quantum (low-temperature) one. The Markovian approximation used in this section [cf. (4.1)] is applicable in the thermal case only. In this (the usual) case, the destruction (delocalization) of quantum correlations occurs far faster than the energy dissipation [38, 263, 146]; that is why we have omitted the frictional force in the equations of motion (4.3), (4.4). Here, Λ in [119] becomes $\lambda^2/2h^2$; $\alpha\lambda h^2$ from [88] becomes $2\lambda^2$ for us. The quantity $\mathcal{D}$ from [41] becomes $h^2/2\lambda^2$, and π in [263], and γ from [262] is denoted here by Γ/m.

5 In Search of Alternative Theories

The tenacious problems of the foundation of quantum theory could be very easily and naturally solved if quantum dynamics enabled one to transform pure states into mixed ones. Hamiltonian dynamics alone, of course, cannot do it. Is Hamiltonian dynamics only an approximation to reality? There is no experimental evidence indicating that, but one finds a hint of the missing Hamiltonian features already in the existence of the classically described macroworld. Hypothetical non-Hamiltonian features corresponding to quantum dynamics are limited by the experimental facts.

Corrections keeping within these limitations can be quite sufficient to prevent the penetration of quantum coherence into the macroworld. The freedom to construct such theories as an alternative to the conventional Hamiltonian theory will exist until all kinds of experiments on macroscopic quantum coherence become available. Usual macroscopic degrees of freedom are presently within the range only of observations for which the interaction with thermal degrees of freedom which introduces fluctuations is sufficient for the quantum coherence to be suppressed (see Sections 3 and 7). The perturbations which follow the usual macroscopic measurements in virtue of the uncertainty principle are imperceptible against the background of thermal fluctuations. And hence there remains a full range for hypotheses on "spontaneous measurements" as yet unknown, which take place permanently and quite independently of human activity. In this case, more or less physical reality can be attributed to "instruments" making these spontaneous measurements; they can be treated as a mathematical abstraction; or, alternatively, they can be identified with the observable classical macroworld as unique reality, interpreting the quantum state as a mathematical abstraction. We know of three such hypotheses.

First, in 1986 Ghirardi, Rimini and Weber[88, 87] advanced a hypothesis on a "spontaneous localization" of particles, i.e. spontaneous measurements of the particle coordinates occurring with a finite accuracy at definite moments to be selected randomly. The results of these spontaneous measurements, "centers of spontaneous localization," form a discrete set of spacetime points which is nothing but a "history" of the classical world. For a single particle, the spontaneous localizations occur astronomically rarely (and hence usually escape observation); however, the path of a macroscopic body is recorded by dense clusters ("galactic," as expressed by Bell[16]) of spontaneous localization centers, which form its classical determinacy. This approach is mathematically worked out on the level of "effects" and "operations," that is a well-known way of a formal description of the quantum/classical relationship [35]. A relativistic field modification, "continuous spontaneous localization," is investigated[86, 64]. For a solid body intensity of momentum fluctuations which are caused by the "spontaneous localizations" of its particles is connected by a definite universal relation with the intensity of angular-momentum fluctuations of this body (caused by the same point); see [67]. This relation is not satisfied for the thermal fluctuations due to the body's interactions with the environment.

Secondly, in 1987 Diosi[69] suggested solving the problem of a quantum measurement in common with that of gravitational-field measurability. The question of the limits of the gravitational-field's measurability and of whether it must in principle be quantized has been discussed for many decades (see, e.g., [32, 206, 197, 233]), but now seems debatable. Diosi maintained that the gravitational field is, in principle, not subject to quantizing, and its interaction with quantized matter affects the state of matter as noise whose intensity corresponds (in the order of magnitude) to the limits of gravitational-field measurability, which can be estimated by thought experiments. Here there is still no study on the level of "effects" and "operations." Furthermore, only the nonrelativistic mechanics of massive particles has been considered, the order of magnitude of the predicted effects being expressed through the combination of gravitational and Planck constants. In this quantitative determinacy the given hypothesis is advantageously distinguished from that on spontaneous localization introducing new physical constants, yet beyond an experimental determination. Note that Diosi's fluctuations are qualitatively distinguished from the thermal fluctuations caused by the interaction of a massive particle with the gravitational finite-temperature field.

The third hypothesis historically preceeds the two mentioned above, but was recently rejected as erroneous. The quantum "evaporation" of black holes predicted by Hawking in 1975 leads to the assertion that at finite time some degrees of freedom lose forever any relationship with the world on this side of the event horizon. Thus, our entire world proves to be a nonclosed system, and a pure initial state goes into a mixed final one. Developing the ideas of quantum gravity, Hawking has arrived at the notion of the virtual process of creating and "evaporating" black holes on the Planck spacetime scale. On a not so minute scale this gives rise, according to [102], to non-Hamiltonian dynamics for the quantum fields. This question was discussed, in particular, in [73, 14, 104, 164]. However, one has good reason to reject the conclusion on non-Hamiltonian dynamics, as obtained by an illicit combination of quantum and classical ideas[49, 89, 103] (however, see [237] for an opposing opinion).

It is interesting that the hypothesis on spontaneous localization predicts gas self-heating [88]. This self-heating occurs at a rate $\sim 10^{-22}$K/sec for those parameters which were selected by the authors of [88]. Diosi's gravitational fluctuations also give rise to heating of a body, its rate being $\sim hGk_B^{-1}\rho$, where ρ is the density of the body. For instance, when $\rho \sim 10^3$kg/m^3, one obtains a rate 10^{-18}K/sec. The corresponding thermal fluctuations are sufficient to suppress the quantum coherence in the macroscopic motion of the body. It is, however, clear that it is extremely difficult to detect so slight a heat release. It is possible that the scattering of starlight on the noncoherent gravitational Diosi's fluctuations might be a more valuable source of observational limitations.

Will the results presented in Section 1 remain valid if the mentioned hypotheses give rise to some non-Hamiltonian quantum theory, which, say, replace the traditional one? The question is one of the kinematics and dynamics of correlations within the scope of the general principles of quantum theory. A positive answer seems evident for kinematics, since only dynamics is affected by the probable change. However, the formulation of the question in operator terms as in Section 1 is believed to be inadequate for a theory with "spontaneous measurements." The question must be one of probabilities of "objective events," and hence of correlations between the results of "spontaneous" (rather than "external") measurements, and this is concerned with dynamics.

We believe that the answer is positive both in kinematics and dynamics, i.e. that the "spontaneous fluctuation" theories cannot go beyond the scope of the class of behaviors determined by (1.9), and hence do not enable the quantum analogs of Bell's inequalities to be violated. Indeed: first, each quantum measurement given in the form of "effects" and "operations" can be represented as an indirect one by means of expanding the Hilbert state space by a new tensor factor describing the device (considered here only as mathematical abstraction), as was shown by Kraus[151]. The device initially was in the prescribed pure state. Its interaction with the object is described by the unitary operator in the tensor product. Further, the prescribed ideal measurement is performed on the device. Second, there is the possibility of localizing the mentioned construction by assuming that each small region of spacetime is associated with its own device which is connected in some way to "our" spacetime, interacts with the known fields within a proper small region, and afterwards is disconnected. We do not know a well-developed theory of such a kind, but in [106, 236] some results were obtained. Local devices which attach to, and then detach from, our Universe resemble Hawking's process (semiclassical rather than virtual). Apparently any spontaneous measurement (or spontaneous fluctuations) theory permits representation via the universal wave function in a definite expanded spacetime, which can have "poor" metric and topological properties. However, as is the conventional Minkowski spacetime, this expanded spacetime is still partially ordered by the chronological relation "earlier/later." Such a theory cannot go beyond the scope of the class of quantum behaviors.

6 Relationship to the Problem of the Foundations of Statistical Physics

Prominent in the method of obtaining the classical behavior within quantum theory, presented in Sections 4 and 5, was a stat-physical description of the environment of a macroscopic quantum system. It is natural that the question of deducing the stat-physical ("secondary") description from the purely dynamical ("primary") one leads to a fundamental problem of theoretical physics, i.e., that of the foundations of statistical physics. Since it is not possible to go into this problem here, we shall limit ourselves to essential results and comments, referring the reader to the literature for more details[191, 152, 239, 240, 201, 141, 137, 138, 135, 133, 129, 125].

Though this problem is often discussed in the context of classical mechanics,[16] we, judging by the principle of "quantum description presumption," discuss this problem as embedded in the quantum theory.[17] This approach originates from a pioneering work of Pauli[191], with further developments particularly in the works of Krylov[152], Van Hove[239, 240], Prigogine *et al.* (see bibliography in [201]), and one of us (Khalfin)[141, 137, 138, 135, 133, 129, 125]. Specifically, the problem of statistical physics arises among the general problems of the quantum theory is as follows.

Let H be a Hamiltonian operator of a closed conservative dynamical system independent in time. Consider the following nonstationary Cauchy problem:

$$\begin{aligned} H|\psi(t)\rangle &= ih\frac{\partial|\psi(t)\rangle}{\partial t} \\ |\psi_0\rangle &= |\psi_{t=0}\rangle \\ \langle\psi_0|\psi_0\rangle &= 1 \end{aligned} \tag{6.1}$$

where the only dependence on time is isolated in the state vector[153]. Let A be the Hermitian operator which defines the problem of the statistical physics of interest, that is, the associated statistical collective (ensemble):

$$A|\xi_k\rangle = a_k|\xi_k\rangle,\ \langle\xi_k|\xi_n\rangle = \delta_k^n \tag{6.2}$$

For simplicity, we will, without loss of generality, consider the operator A with a discrete spectrum. The goal of statistical physics is the description of the set of probabilities $\{P_k(t)\}$:

$$\begin{aligned} P_k(t) &= |p_k(t)|^2 \\ p_k(t) &= \langle\xi_k|\psi(t)\rangle = \langle\xi_k|e^{-iHt/h}|\psi_0\rangle \end{aligned} \tag{6.3}$$

If the set $\{P_k(t)\}$ is known, all usual questions of statistical physics can be answered; in particular, $\{P_k(t \to \infty)\}$ will determine an equilibrium distribution. If one could succeed in showing, by solving the corresponding

[16] The most advanced results for classical billiard systems were obtained in the work of Sinai[221].

[17] In the context of quantum theory, the primary probability structure of statistical physics does not resemble the hidden deterministic behavior (see Section 1) which is assumed in discussing the problem of the foundations of statistical physics in classical theory.

Cauchy problem (6.1), that the $P_k(t)$ obeyed the balance equations ("master equations," Chapman-Kolmogorov equations) of a Markov chain

$$\frac{dP_k(t)}{dt} = \sum_n [P_n(t)W_{nk} - P_k(t)W_{kn}] \tag{6.4}$$

where W_{nk} (W_{kn}) are the probabilities of the transitions $n \to k$, ($k \to n$) in unit time, which satisfy the condition of dynamical reversibility $W_{nk} = W_{kn}$ and are positive, the problem of the foundation of statistical physics would be solved once and for all. As proven in probability theory[45], all basic predictions of axiomatic statistical physics are valid for such Markovian chains (processes) (irreversible entropy-increasing tending to the equilibrium state at $t \to \infty$, ergodicity, intermixing, exponential decreasing of correlations at $t \to \infty$, the standard description of the equilibrium distributions, and so on). A positive solution of the problem of the foundations of statistical physics would be an important contribution, in that statistical physics correctly describes an immense number of different physical phenomena—from thermodynamics and hydrodynamics to the kinetics of chemical reactions, processes of self-organization, and so on. Such a solution can only take place with definite limitations on the operators H, A and the initial vector $|\psi_0\rangle$. At first sight, the problem of obtaining irreversible behavior from the quantum time-reversible theory appears to be a basic impediment to obtaining the predictions of statistical physics. However, we recall that methods are already available to describe the time-irreversible decay of unstable states (elementary particles, nuclei, atoms and so on) in the scope of time-reversible quantum theory.

This analogy with the quantum decay theory underlies the approach to the problem of foundations of statistical physics in the works of one of us (Khalfin)[141, 137, 138, 135, 133, 129, 125]. The fact is that there exist solutions behaving irreversibly among the nonstationary solutions of the Cauchy problem for a time-reversible Schrödinger equation (6.1), i.e., there is a possible spontaneous violation of reversibility (t-invariance) in quantum theory. We have

$$\begin{aligned}
p(t) &= \langle\psi_0|\psi(t)\rangle = \langle\psi_0|e^{-iHt/h}|\psi_0\rangle \\
&= \sum_k |c_k|^2 e^{-iE_k t/h} + \int_{\mathrm{Spec(H)}} |c(E)|^2 e^{-iEt/h}\, dE \\
H|\phi_k\rangle &= E_k|\phi_k\rangle,\ \langle\phi_k|\phi_i\rangle = \delta_k^i \\
H|\phi_E\rangle &= E|\phi_E\rangle,\ \langle\phi_{E'}|\phi_E\rangle = \delta(E'-E) \\
|c_k| &= |\langle\phi_k|\psi_0\rangle| = |\langle\phi_k|\psi(t)\rangle| \\
|c(E)| &= \langle\phi_E|\psi_0\rangle| = |\langle\phi_E|\psi(t)\rangle|
\end{aligned} \tag{6.5}$$

on the basis of the fundamental Fock-Krylov Theorem[153]. In order to have solutions $|\psi(t)\rangle$ that behave irreversibly, i.e., such that

$$\lim_{t\to\infty} |p(t)| = \lim_{t\to\infty} |\langle\psi_0|\psi(t)\rangle| \ = \ 0 \tag{6.6}$$

it is necessary and sufficient[153], as follows from (6.5), that: (a) the operator H should have an absolutely continuous spectrum, and (b) the initial vector $|\psi_0\rangle$ (and hence $|\psi(t)\rangle$) should have no weight ($c_k = 0$ for all k) on the discrete spectrum. In this case, it follows from (6.5) that

$$p(t) = \int_{\mathrm{Spec(H)}} |c(E)|^2 e^{-iEt/h}\, dE \tag{6.7}$$

where $\omega(E) \equiv |c(E)|^2$ is the density of the energy distribution (invariant of the motion) for the nonstationary quantum system considered. But if some $c_k \neq 0$, then there follows at once from (6.5) a quantum analog of Poincaré's recurrence theorem. If, as is usually done, a quantum-mechanical system of a finite number of particles interacting through some potential in a finite volume is considered as a physical system of statistical physics, then condition (a) above is evidently violated, and there exist no solutions that behave irreversibly because of the quantum analog of the Poincaré's recurrence theorem. However, such a quantum mechanical potential model is obviously approximate from the point of view of quantum field theory, and there exist solutions behaving irreversibly if we give up this approximation. The operator A can be used for a finite number of particles in a finite volume, which particles determine a statistical mechanical ensemble. However, the interactions of particles with each other and with the walls of the volume $\mathcal{V}$ are given through the interaction of the fields given in a infinite space, the quanta of which are not introduced into the description of the operator A, but which are important for the operator H. If we take into account the quantum field which gives an absolutely continuous spectrum to the operator H, we obtain, according to (6.6), the irreversibly behaving solutions which solve the problem of irreversibility. However, to choose such irreversible solutions which correspond to the problems of statistical physics, one must select definite initial vectors in accord with the operators H and A. The subsequent presentation is essentially concerned with taking into account the spectral principle of the quantum theory, i.e., the assumption of the boundedness from below of the spectrum. Taking, without loss of generality, the vacuum state energy to be zero, one obtains

$$\begin{aligned} p(t) &= \int_0^\infty \omega(E) e^{-iEt}\, dE \\ p_k(t) &= \int_0^\infty c^*(E)\alpha_k(E) e^{-iEt}\, dE \\ \alpha_k(E) &= \langle\phi(E)|\xi_k\rangle \end{aligned} \tag{6.8}$$

As shown in [141, 144, 142], the condition of spectrality leads to analytical properties of the amplitudes $p(t)$, $p_k(t)$ in the complex-t half-plane, from which follows a violation of the Markovian property, because $p(t)$, $p_k(t)$ have nonexponential terms[141, 144, 143, 142] which are irremovable for any initial vector $|\psi_0\rangle$. This implies the nonexistence of transition probabilities per unit time which are independent of time, i.e., it implies a nonhomogeneity in time. Nonexponential terms, as shown in [141, 144, 143, 142], are analytic functions in the complex-t right half-plane $\mathrm{Re}(t) > 0$. Therefore, they are nonzero in any time interval. However, they are important (larger than the exponential Markovian type terms) only for very small or very large times, and are unimportant for "intermediate" times. For example, let we choose definite interrelated choice[18] of H, A such that[141, 129]

$$\begin{gathered} \langle \phi_{E'} | A \phi_E \rangle = b(E')\delta(E' - E) + g(E', E) \\ c(E) \approx (E - E_0 - i\Gamma)^{-1} \end{gathered} \tag{6.9}$$

where $b(E')$, $g(E', E)$ are sufficiently smooth functions of their arguments. For such a choice, the main (exponential) term in $p_k(t)$ gives rise to the Markovian features, and hence to the description according to the axioms of statistical physics, so that Eqs. (6.4) are satisfied. At the same time, the nonexponential terms (principally irremovable) lead to differences, in the domain of quantum theory, from the predictions of the usual axiomatic statistical physics: violation of Markovian properties, "infinite" memory, violation of ergodicity, intermixing, nonexponential decrease of the correlation functions, deviation of the equilibrium distributions from the standard ones (the dependence on the relaxation time), and so on. In this case, the influence of the nonexponential terms proves to be non-uniform for various effects.

Now, the problem of the foundations of statistical physics in the domain of quantum theory received, contrary to widespread expectations, a negative solution. Generally speaking, the predictions of the usual axiomatic statistical physics do not follow from the quantum theory. However, for conventional physical systems, the basic predictions of the usual axiomatic statistical physics give a very good approximation for the "intermediate" times usually considered. Thus, the nonexponential (non-statistical-physical) terms have the order $\mathrm{o}(\mathcal{N}^{-2})$ (where $\mathcal{N}$ is the num-

[18] The conditions thus formulated in the spectral language do not explicitly depend on the number of particles, which makes difficult their testing for real systems. On the other hand, these conditions can be fulfilled for some systems having a small number of particles which behave for moderate times in accordance with the usual axioms of the statistical physics.

ber of particles) for a sufficiently weak interaction responsible for the nonequilibrium properties (instability). Therefore, for systems with a sufficiently great number of particles, the basic predictions of statistical physics are fulfilled to a tremendous—practically infinite—accuracy. At the same time, the difference between the predictions of the quantum theory and those of statistical physics can be important for some "delicate," "deep" problems. These differences have already been observed experimentally[75]; for example, the corresponding velocity correlation functions decrease nonexponentially in spite of a perfectly well fulfilled diffusion law (Einstein law for a Brownian particle).

The well-known works of Van Hove, Prigogine, and others dealt with the quantum-mechanical potential model, so that the irreversibility was accomplished at the expense of taking the thermodynamic limit

$$\begin{aligned} \mathcal{N} &\to \infty, \\ \mathrm{V} &\to \infty, \\ \mathcal{N}/\mathrm{V} &= \text{const}, \end{aligned}$$

and the equations of Markovian type (6.4) were deduced by selecting definite $U, H = H_0 + U$, $A = H_0$, $[H_0, U] \neq 0$ and $|\psi_0\rangle$ in the additional two-dimensional limit

$$\begin{aligned} \lambda &\to 0, \\ t &\to \infty, \\ \lambda^2 t &= \text{const} \end{aligned}$$

This additional limit makes $H = H_0 + \lambda U$ explicitly depend on time (since $\lambda = \lambda(t)$), which is unsatisfactory from the point of view of first principles. It was shown (in [141, 137, 138, 135, 133, 129, 125]) that all nonexponential ("nonstatistical-physical") terms in this limit vanish, which explains how one obtains the Markovian "master equation" in [239, 240, 201].

7 Experimental Tests of the Universal Validity of the Quantum Theory

It was shown in the preceding sections that the classical Bell inequalities and their quantum analogs permit the immediate comparison of various physical theories, or to be precise, of the fundamental principles of these theories. This makes it possible to compare such qualitatively different concepts as the classical (including a hidden deterministic one in terms of Section 1) and the quantum concepts of the physical world.

The classical Bell inequalities, as is well known, evolved from discussions of thought experiments (now real experiments[47, 8, 9, 10, 198]) on observing quantum correlations at macroscopic distances, originally presented in the famous Einstein, Podolsky, and Rosen (EPR) paper[70]. In the EPR work, coordinate-momentum observables[19] (with continuous spectrum) were used for the correlation experiments. On the other hand, Bell's inequalities in Bell's pioneering work[22] are related to the spin (with discrete spectrum) version of the EPR experiment, which was proposed by Bohm[25, item 22.16]. Recall that the spin experiment enables us to select the correlation functions in such a way[47] that the quantum theory predicts the extremal value equal to $2\sqrt{2}$. This makes the experiment a distinguished one insofar as the difference in the predictions of the classical and the quantum theories cannot be larger in any other correlation experiment. As shown in Section 1, the quantum analogs of Bell's inequalities are limited by the same extremal value $2\sqrt{2}$; therefore, the spin experiment can be also regarded as testing the quantum analogs of Bell's inequalities. Among the real experiments for testing Bell's inequalities (for more information on these experiments see [8, 9, 10]) the most accurate are equivalent to "Bohm spin" experiments using the polarized photons in the cascade decay of excited atomic levels. Of course the real experiments are accompanied by some technical (less than 100% efficiency of the photodetectors, and so on) and non-technical problems (see [7, 112], and [180, 181, 83, 182, 111, 190]). For those reasons, such experiments are not quite "pure" from the theoretical point of view. In any case, the undoubted violation of the classical Bell inequalities (or Bell-CHSH inequalities) and the confirmation of the quantum mechanical predictions in the experiments[8, 9, 10, 188, 219], in spite of their nonideality, can be considered as the unconditional death of all admissible alternatives (the local theories with a hidden parameters) which Einstein had sought.

The quantum analogs of Bell's inequalities, which were first derived in [51], are model-independent predictions from the first principles of the quantum theory, of the same generality as the classical Bell inequalities for the classical theory. Therefore, the experimental testing of the quantum analogs of Bell's inequalities is as necessary in the new area of higher energies and for new classes of physical objects (and not only for new elementary particles) as the experimental testing of other fundamental

[19]The correlation experiments are discussed from the point of view of the corresponding Bell inequalities with the use of the original EPR formulation through the coordinate-momentum observables in recent works[147, 113, 43, 18, 93].

predictions of the theory, such as the famous conservation laws. It is important to note that the quantum analogs of Bell's inequalities are nontrivial, because, as was shown in Section 1, these inequalities (and all the more so the classical Bell inequalities) can be violated for the class of the stochastic behavior.

As mentioned above, it follows that the experimental testing of the classical Bell inequalities for the extremal spin version will be also tested at the same time as the quantum analogs of Bell's inequalities. But real experiments with polarized photons in the decay cascade of atomic levels [8, 9, 10], and also experiments like [188, 219], are unlikely to be of interest from this point of view. It is difficult to believe that the validity of the quantum concept would be violated in investigations of low-energy photons in the decay of the exciting atomic levels.

Experiments with K°-$\bar{K}^\circ$, D°-$\bar{D}^\circ$, and B°-$\bar{B}^\circ$ mesons, and with their possible future analogs for the extremal energies of existing accelerators and for higher energies in the future (like the SSC accelerator) are of evident interest. From this point of view, experiments with these mesons for testing classical Bell inequalities were discussed in [165, 222, 217, 134, 55]. In [134], such experiments were discussed from the point of view of the quantum analogs of Bell's inequalities. Specifically, the experiments with storage rings like $e^+e^- \to \phi \to K^0\bar{K}^0$, $e^+e^- \to \psi(3700) \to D^0\bar{D}^0$, $e^-e^+ \to \gamma(4s) \to B^0\bar{B}^0$ in the meson factories were discussed. For the description of the time evolution of K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, B^0-$\bar{B}^0$ mesons, in order to take into account the CP-violation it is necessary to use a theory[140, 139, 136, 132, 130, 126, 127, 123] beyond the Weisskopf-Wigner approximation, as in the usual theoretical consideration[166].

The experimental investigations of macroscopic quantum effects were induced by theoretical works of Leggett *et al.*[36, 37, 38, 170, 171], in which were discussed both the possible experiments on macroscopic quantum tunneling (MQT) and macroscopic quantum coherence (MQC). MQT effects for the macroscopic degree of freedom—the phase-order parameter in the Josephson transition—were experimentally investigated (see [183, 90] and other works mentioned in [90]). The results of these experiments for sufficiently low temperatures are well explained by the corresponding MQT effects. But, as was noticed by the authors of those works, such experimental results are not unconditional proof of the existence of MQT effects, because it is possible, in principle, to find another explanation due to some other effects, which are not taken into account in the usual theoretical consideration. One such possibility was discussed in recent work[225]. In connection with this, it will be very interesting to repeat the experiments of [90] (and the analogous experiments of other works mentioned in [90]) but with the new high-temperature superconductors.

In fact, if the results of the experiments of the type of [90], but with high-temperature superconductors, are the same as for the ordinary (low-temperature) superconductors, it will strongly suggest that the previously observed effects[90] are not real macroscopic quantum effects (MQT), but are some new specific effects of the type investigated in [225]. This statement follows from the fact that real macroscopic quantum effects are possible only for sufficiently low temperature and cannot exist for the high temperatures which characterize modern high-temperature superconductivity. At the same time, if these effects are not observed, it will mean that the previously observed effects are real macroscopic quantum effects (MQT).

It is also necessary to mention that theoretical results which were derived in [36, 37, 38, 170, 171] and in many subsequent studies (cf. [170]) and which describe MQT effects, are based on using the Euclidean approach (Langer-Polyakov-Coleman instanton method[163, 199, 39, 50]) in the evaluation of the corresponding path integrals. But as was recently proven ([131, 128, 124]) by one of us (Khalfin), unfortunately this method is in the best case without proof, and for some of the more interesting cases (like the asymmetrical potential with two walls) false.

Real experiments on MQC up to now have not been performed.

Now we discuss possible experiments connected with macroscopic quantum effects which follow from the theoretical considerations in Sections 3 and 4 and which do not have the defects mentioned in connection with other macroscopic effects (see above). If the time of the classical factorization τ_f is sufficiently small, then the commonly investigated "intermediate" times t are bigger than τ_f—and for this reason the exact classical description of the dynamics of the corresponding physical systems will be true. But if τ_f is sufficiently large then the usual "intermediate" times will be less than τ_f, and this gives the possibility of seeing macroscopic quantum effects in the time evolution of the corresponding physical systems. It is essential that the time of the classical factorization was defined not only by the parameters of the considered physical system but also by the parameters of the environment (temperature, friction force, pressure of the gas, and so on). By choosing various values of these environment parameters for the same physical system, it will be possible to arrange that the corresponding time of the classical factorization will be small so that the dynamics will be exactly classical and macroscopic quantum effects cannot be present under these conditions. For a different set of environmental parameters the time of classical factorization will be large and then macroscopic quantum effects will be observable in the dynamics. We shall give only one concrete example, without going into technical details (see [146]).

Let us investigate a solid body with linear dimension l, immersed in a rarefied gas at temperature T and in thermal electromagnetic radiation with the same temperature. The intensity of the fluctuation noise for the translational degrees of freedom can be estimated by the formula

$$\lambda^2 \sim l^2[\rho m_0^{-\frac{1}{2}}(k_B T)^{3/2} + ch^2 l_{\text{therm}}^{-9} \min(l^4, l_{\text{therm}}^4)] \qquad (7.1)$$

where ρ is the density of the gas, m_0 the mass of the gas molecules, $l_{\text{therm}} \sim ch/k_B T$ the characteristic wavelength of the thermal radiation, and λ the value which was introduced in (4.1). Formula (7.1) is derivable by direct calculation[119] and also via the fluctuation-dissipation relations[146]. If we know λ, it is possible to define τ_{fluc} [see (4.2)], which is closely connected with τ_f [see (4.29)]. For a body of size $\sim$1 cm and mass $\sim$1 g, within a rarefied gas with density $\sim 10^{-26}$ kg/m^3 and temperature $\sim$1 K and thermal radiation at the same temperature as was derived in [146], the estimate $\tau_{\text{fluc}} \sim 10^6$ sec clearly gives a macroscopic time! For the rotational degrees of freedom the dissipation can be even smaller. But the translation degrees of freedom, in contrast to the rotational degrees of freedom, are easy to use for the construction of nontrivial dynamical systems with small dissipation by using the Coulomb interaction. In this case new sources of dissipation play a role: variable deformations lead to nonuniform heating, thermal exchange, and so on. The estimates of these effects, which were derived by us[146] for a sapphire crystal starting from experimental data[30], gave $\tau_{\text{fluc}} \sim 10^5$ sec when the Hamiltonian dynamics associated with Coulomb interaction was characterized by the time $\tau_{\text{mech}} \sim 1$ sec. In light of the results of Section 4 [see (4.34)], this presents the possibility of a dynamical origin of quantum correlations between crystals. In [146] we do not estimate the Ohmic dissipation which follows from the nonzero sapphire conductivity. In reality these losses are as important as the corresponding electromagnetic forces of Nyquist noise, which cause fluctuations of the electromagnetic field in the near zone around a crystal. However, these losses could be isolated by a thin metal casing (cover) around each crystal.

Thus, if we are very careful, so as to practically realize the isolation of collective degrees of freedom from the thermal ones, quantum correlations can appear dynamically due to the mechanical motion of macroscopic bodies and as a result of macroscopic interactions, and they can exist for a macroscopically long time. In this way macroscopic quantum effects become possible. An experimental observation of these effects would allow us to disprove alternative variants of the quantum theory that have been considered in Section 5.

The experimental observations of macroscopic quantum effects could essentially change and add to our notion (and understanding) of the surrounding macroscopical phenomena, which up to now we have described (and understood) as purely classical phenomena. Likewise, modern biology (biochemistry, biophysics) and computer logic have been based on classical ideas (even quantum computers of the type of [76] entirely realize a classical logic). The results of quantum theory are used there only in an auxiliary role, that is, for calculation of the corresponding classical parameters. The observation of real macroscopic quantum effects could make possible quite new applications (for example, a new logic of pure quantum computers), and also the understanding, within the quantum theory, of the strange problems of biology, for which wholeness (indivisibility) is so typical—as well as for specific quantum phenomena. Moreover, such macroscopic quantum effects may be the consequences of very deep microscopic quantum phenomena, which are connected with the physics of elementary particles.

8 Conclusions

We have made the next attempt of reconstructing classical reality as a superstructure over quantum dynamics. The difference from early works [185, 74, 244, 52, 107] is the use of dissipation, which is typical of works of the last decade [168, 36, 263, 245, 65]. The newness of our way of posing the question is the use of ideas originating in Bell's inequality. The newness of our results is a sharp boundary for the domain of existence of classical reality. Besides this, we endeavor to give forerunners their due, and stimulate further investigations by way of elucidating the context.

It would be worth achieving a better understanding of what collective and thermal degrees of freedom are, and what dissipation is. Usually one begins with two individual systems, one (mechanical) having few degrees of freedom, the other being a reservoir (heat bath), and then introduce an interaction between them, resulting in dissipation. In so doing, some degrees of freedom of the reservoir "dress" the given "bare" mechanical degrees of freedom, forming collective ones. It would be worthwhile to be able to find collective degrees of freedom in a given whole system. Note that in reality each collective degree of freedom involves all fields (including gravitation). In this respect the "collective approach" is opposed to the "electromagnetic approach" [226] as well as the "gravitation approach" [69].

Why we do not expect the notion of classical reality to admit the only true, short, exact and clear definition, even in principle, was explained in Section 3. It may well be true for the notion of collective degrees of freedom, as well.

Though we proclaim, in principle, that the quantum dynamics is taken as it is, without any approximation, in practice, we apply an approximation, taking the relaxation time of the thermal environment as infinitesimal, and hence the evolution of the mechanical system as Markovian. Of course, this is a drawback. We think that it will be possible to generalize our approach to a non-Markovian case, but the proposed mathematical technique will need a re-making.

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