
Consider the following transition matrix. Which states are recurrent and which are transient? Give reasons for your answers.

$$\begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0 & 0.3 & 0.4 \end{pmatrix}$$

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 If $x \rightarrow y \not\rightarrow x$ (for some y) then x is transient. If $x \rightarrow y$ implies $y \rightarrow x$ (for all y) then x is recurrent (provided that all states are a finite set).

The states 2 and 4 are recurrent, since $2 \leftrightarrow 4$ and the set $\{2, 4\}$ is closed. The states 1, 3 and 5 are transient, since $3 \rightarrow 1 \rightarrow 2$ and $5 \rightarrow 2$.

Three of every four trucks on the road are followed by a car, while only one of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

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 Assuming the Markov property, we have transition probabilities $p_{\text{truck,car}} = 0.75$ and $p_{\text{car,truck}} = 0.2$. The stationary distribution satisfies $p_{\text{truck}} \cdot 0.75 + p_{\text{car}} \cdot (1 - 0.2) = p_{\text{car}}$ and of course, $p_{\text{truck}} + p_{\text{car}} = 1$. It follows that $p_{\text{truck}} = 4/19$.
