After the exam of 11.07.2014: Typical errors, comments etc.

Question 1

Error: "$\frac{|\sin\frac{1}{x}|^{1/n}}{\mathrm{e}^x\sqrt{x}} \uparrow \frac{1}{\mathrm{e}^x\sqrt{x}}$ (as $n \to \infty$) for all $x \in (0,\infty)$".

Clarification: this relation fails for $x = \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \dots$

Penalty: 20 points.

Error: "$\int_\delta^R f_n \to \int_\delta^R f$ whenever $0 < \delta < R < \infty$, therefore $\int_0^\infty f_n \to \int_0^\infty f$".

Clarification: additional arguments are needed; in full generality this relation may fail; a counterexample: $f_n = 1\!\!1_{[n,n+1]}$, $f = 0$.

Penalty: 7 points.

The same penalty applies to unexplained transformations of the form "$\lim_{\delta,R} \lim_n \dots = \lim_n \lim_{\delta,R} \dots$" etc.

Question 2

Item (d): within the disk $\big(x-(r+s)\big)^2 + y^2 \le r^2$ the curl must be $O\big(\frac{1}{s^3}\big)$ but need not be $O\big(\frac{1}{(r+s)^3}\big)$.

Question 3

First, the relation "$F(x,y,z) = o(x^2+y^2+z^2)$ as $x^2+y^2+z^2 \to \infty$" does not imply "$F(x,y,z) \to 0$ as $x^2+y^2+z^2 \to \infty$".

Second, F vanishes outside the set $\{(x,y,z) : 0 \le (x^2+y^2)^{3/2} z \le 1\}$ and therefore, by continuity, F vanishes also on the boundary of this set. In other words, F vanishes outside the open set $\{(x,y,z) : 0 < (x^2+y^2)^{3/2} z < 1\}$.

Third, this unbounded open set has two "escapes to infinity". One escape: $z \to +\infty$, $x^2+y^2 \to 0$. The other escape: $z \to 0+$, $x^2+y^2 \to \infty$. Regretfully, many students concentrated on the former escape only, and did not succeed, since the key to the solution is the latter escape. As a remedy, 20 points are given for every (correct) proof that the flux through the plane $z = c$, $c > 0$ does not depend on c (even though this is a wrong way).

Question 4

It is tempting to "project" the tangent line to the given manifold M (by taking the closest point of M). However, is this projection a continuously differentiable mapping? We have no means to prove this. (Not even that the closest point is unique.) This is a wrong way.

General remark

A factor 1.1 is given because of the abnormal situation we face these days.

Grades statistics

Total (with factor)	Total (no factor)	Question 1	Question 2	Question 3	Question 4
114	104	34	40		30
110	100	30	40		30
108	98	28	40		30
94	85	35		20	30
91	83		33	20	30
88	80		30	20	30
86	78	28		20	30
86	78	28	30	20	
83	75	30		15	30
75	68	33	30		5
70	64	34	20	10	
69	63	33		0	30
68	62	35	17		10
66	60	35		0	25
66	60	35		20	5
64	58	28		0	30
61	55	35	20	0	
55	50		20		30
55	50			20	30
47	43	15	28		0
39	35	15		20	0
37	34	15	19		0
33	30			0	30
28	25	15	10		0