After the exam of 26.01.2014: Typical errors, comments etc.

Question 1

Item (a)

Error: "If $g(x) \neq 0$ then either $g(x) > 0$ (and $f(x) = g(x)$) or $g(x) < 0$ (and $f(x) = -g(x)$)".

Clarification: $g : \mathbb{R}^n \to \mathbb{R}^m$ rather than $\mathbb{R}^n \to \mathbb{R}$.

Penalty: 7 points.

The same penalty applies to other arguments that make sense only for $m = 1$.

Item (c)

Error: "define h by $h(x) = \big(f(x)\big)^2$, then $\nabla h(x) = 2f(x)\nabla f(x)$; using ∇f found in Item (a) we see that ∇h vanishes on Z_g".

Clarification: the formula for ∇f does not apply on Z_g. If you really want to go this way, you have to prove two claims for $x_0 \in Z_g$. First, $\nabla h(x_0) = 0$. Second, $\lim_{x\to x_0} \nabla h(x) = 0$.

Penalty: 5 points.

Comment: It is much easier to use the C^1 function $y \mapsto |y|^2$ on $\mathbb{R}^m$.

Question 2

Fatal error:[1] "a point of local extremum of $\varphi_a + \varphi_b$ on $S_1(0)$ is also a point of local extremum of $\sin \frac{1}{2}\varphi_a + \sin \frac{1}{2}\varphi_b$ on $S_1(0)$".

Clarification: The gradient of $\varphi_a + \varphi_b$ is generally not collinear to the gradient of $\sin \frac{1}{2}\varphi_a + \sin \frac{1}{2}\varphi_b$.

Fatal error: "$\nabla f = 0$ at a point of local extremum of f on $S_1(0)$".

Clarification: rather, $\nabla f = \lambda \nabla g$; λ need not vanish.

Fatal error: long calculations that do not prove that x is a linear combination of a, b.

Clarification: You need to prove that x is a linear combination of a, b; you do not need to calculate explicitly the coefficients of the linear combination! Long calculations are irrelevant; they do not bring you points, and are not checked.

Comment: In particular, you do not need to know that $(\arcsin t)' = \frac{1}{\sqrt{1-t^2}}$; all you need is just $\arcsin \in C^1(0,1)$.

Additional comment: I did not ask you to really find these extrema, but if you want (after the exam) to find them anyway, do it in two steps:

[1] It means, no points for this question!

first, using Lagrange multipliers, prove that all extremal points are situated on the plane spanned by a, b, thus, on a circe (the plane intersected with the sphere);
second, parametrize this circle by a single angular variable and apply the *one-dimensional* calculus.

Question 3

Item (a)

Error: "$\inf_P \big(U(f,P) + U(g,P) \big) \le \inf_P U(f,P) + \inf_P U(g,P)$".

Clarification: Generally, $\sup(X+Y) \le \sup(X) + \sup(Y)$ and $\inf(X+Y) \ge \inf(X) + \inf(Y)$. A specific argument (joint refinement of two partitions) is needed.

Penalty: 5 points.

The same penalty applies to other incorrect treatment of $\inf_P$.

Item (c)

Fatal error:[1] "$v^*(E) + v^*(F) = v^*(E \cup F) + v^*(E \cap F)$".

Clarification: v is additive, but v^* is not.

Fatal error: "${}^*\!\!\int (1\!\!1_E + 1\!\!1_F) = {}^*\!\!\int 1\!\!1_E + {}^*\!\!\int 1\!\!1_F$".

Clarification: $\int$ is additive, but ${}^*\!\!\int$ is not.

Fatal error: "${}^*\!\!\int 1\!\!1_{E \cup F} \le {}^*\!\!\int 1\!\!1_E + {}^*\!\!\int 1\!\!1_F - {}^*\!\!\int 1\!\!1_{E \cap F}$".

Clarification: rather, ${}^*\!\!\int 1\!\!1_{E \cup F} \le {}^*\!\!\int 1\!\!1_E + {}^*\!\!\int 1\!\!1_F + {}^*\!\!\int (-1\!\!1_{E \cap F})$; the last term is $\big(-{}_*\!\!\int 1\!\!1_{E \cap F} \big)$, not $\big(-{}^*\!\!\int 1\!\!1_{E \cap F} \big)$.

Error: "$1\!\!1_{E \cup F} = 1\!\!1_E + 1\!\!1_F$".

Clarification: E, F need not be disjoint.

Penalty: 5 points.

Question 4

Item (a)

Fatal error: $v(E)$ is an expression containing r.

Clarification: E is defined without any parameter; its volume cannot depend on some r.

Error: "E_z is a disk of radius $1 - z$; its area is $\pi(1-z)^2$".

Clarification: no, its radius is $\sqrt{1-z}$ and area $\pi(1-z)$.

Penalty: 6 points.

[1] It means, no points for this item.

Grades statistics

Total	Question 1	Question 2	Question 3	Question 4
110		40	30	40
110	30	40		40
110		40	30	40
110		40	30	40
108		38	30	40
107	30	40		37
100	30	40	30	
100	30	40	30	
100	30	40	30	
100	30		30	40
100	30		30	40
100	30	40	30	
100	30	40	30	
99	29		30	40
98	30		28	40
98	30		28	40
97		40	30	27
95	30	40	25	
95	30		25	40
93		40	23	30
92		40	25	27
91	23		28	40
90	30		30	30
88	23	40	25	
85	30		28	27
81	27		27	27
81	16	40	25	
81	30		30	21
80	30		25	25
80		25	25	30

Total	Question 1	Question 2	Question 3	Question 4
76	30		25	21
76	15	40		21
73	25		25	23
73	22		30	21
71	20		30	21
70	20		23	27
69	17		25	27
64	24	0		40
62	30		5	27
61	10		24	27
60	30	0	30	
60	25		14	21
60		0	20	40
60	30	0		30
60	30	0	30	
60	30	0	30	
57	30	0		27
55	30	0	25	
54	15		18	21
52	25	0		27
52		0	25	27
52		0	25	27
51		0	30	21
51	16		17	18
50	25	0	25	
48	28	0	20	
47		0	20	27
46	20	0	26	
40	22	0	18	
33	12		10	11
32	16	0		16
26	20	0	6	
26	8		5	13
20	10	0	10	
16	8		8	0
10	0	0	10	
8	5		3	0
0	0	0		