Pursuing to the Limit*

Sergiu Hart†

July 2018

QUESTION. At time $t = 0$, particle A is at the origin $(0, 0)$ and particle B is at $(0, 1)$. A moves to the right (on the horizontal axis) with velocity 1 and B moves towards A with velocity 1. What will be the distance between A and B when t goes to infinity?

ANSWER. 1/2.

PROOF. Let θ_t denote the angle that the line through the positions of A and B at time t makes with the horizontal axis.

1. The distance d_t between A and B increases with velocity $\cos\theta_t$ (due to A) and decreases with velocity 1 (due to B).
2. The horizontal distance x_t between A and B increases with velocity 1 (due to A) and decreases with velocity $\cos\theta_t$ (due to B).
3. From #1 and #2 it follows that $d_t + x_t$ stays constant, and so it always equals $d_0 + x_0 = 1 + 0 = 1$.
4. The vertical distance y_t between A and B decreases with velocity $\sin\theta_t$ (due to B); since y_t has a limit it follows that $\sin\theta_t \to 0$, and so $\theta_t \to 0$, which implies that $y_t \to 0$, and thus $d_t - x_t \to 0$.
5. From #3 and #4 it follows that both d_t and x_t converge to 1/2.

*This solves a question asked by Benjy Weiss, who got it as a conjecture from ...

†The Hebrew University of Jerusalem. *email*: hart@huji.ac.il *home page*: www.ma.huji.ac.il/hart