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ON EQUILIBRIUM ALLOCATIONS AS DISTRIBUTIONS ON THE COMMODITY SPACE*

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It is shown that the *distribution of agents' characteristics* is a concise and accurate description of an economy as far as Walrasian equilibrium analysis for large economies is concerned: Let $\mathscr{E}$ be an exchange economy; $W(\mathscr{E})$, the set of Walras allocations for $\mathscr{E}$; and $\mathscr{D}W(\mathscr{E})$, the set of distributions on the commodity space of the allocations in $W(\mathscr{E})$. It is shown that for two atomless economies $\mathscr{E}_1$ and $\mathscr{E}_2$ which have the same distribution of agents' characteristics, the sets $\mathscr{D}W(\mathscr{E}_1)$ and $\mathscr{D}W(\mathscr{E}_2)$ have the same closure. For every distribution μ of agents' characteristics is defined a standard representation $\mathscr{E}^\mu$, and it is shown that $\mathscr{D}W(\mathscr{E}^\mu)$ is closed. Further, the correspondence $\mu \mapsto \mathscr{D}W(\mathscr{E}^\mu)$ is shown to be upper hemicontinuous.

1. Introduction

An economy $\mathscr{E}$ is usually described by a finite set A, the set of economic agents and by specifying for every individual agent a in A his characteristics. For example, in a situation of pure exchange an economic agent is characterized by his preference relation $\precsim$ and his initial endowment vector e. Thus an exchange economy is described by a mapping

$$\mathscr{E} : A \to \mathscr{P} \times R^l_+,$$

when $\mathscr{P}$ denotes the set of all preference relations $\precsim$ defined on the closed positive orthant R^l_+ of the commodity space R^l.

If the set A of agents is small (e.g., $\# A = 3$), then the description of an economy as an assignment $a \mapsto (\precsim, e_a)$ of agents to characteristics seems to be the appropriate one. However, when the set A is large (e.g., $\# A > 1000$) one is tempted to give up this individualistic description and to replace it by a more

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collectivistic view, namely, to consider the *distribution* $\mu_{\mathscr{E}}$ of the mapping $\mathscr{E}$. That is to say, for every subset B in $\mathscr{P} \times R^l$ we consider the number $\mu_{\mathscr{E}}(B)$ which is the fraction of the totality of agents in A having their characteristics in the set B:

$$\mu_{\mathscr{E}}(B) = \#\{a \in A | \mathscr{E}(a) \in B\} / \# A .$$

When we describe an economy by its distribution μ of agents' characteristics, then we can no longer speak of *some particular* agent and *his particular* characteristics; the set of agents is no longer specified. However, we can speak meaningfully of the *fraction* of the totality of agents having their characteristics in a particular subset B in $\mathscr{P} \times R^l$.

Now the question arises: Is it legitimate to describe an economy by a distribution of agents' characteristics, that is to say, does the distribution $\mu_{\mathscr{E}}$ of an economy $\mathscr{E} : A \to \mathscr{P} \times R^l_+$ contain the relevant information?

The answer to this question clearly depends on what type of equilibrium concepts and what kind of problems we want to investigate. For example, if an economy is described by a distribution of agents' characteristics only, then it is not clear how the core of that economy should be defined. The concepts of 'coalition' and 'to improve upon' require the individualistic description of an economy as a mapping which assigns to every individual agent his characteristics. Yet, when we consider distribution of agents' characteristics we have large economies in mind, and for large economies we have the well-known Theorem of Aumann (1964) which says that the core and the set of Walras allocations coincide. Thus, in situations where distributions of agents' characteristics seem to be the natural concept, namely, in large economies, we can concentrate on Walras equilibria.

We shall show in what follows that with respect to Walrasian Equilibrium Theory the distribution of agents' characteristics actually contains the relevant information.

2. Large economies

Firstly, we have to recall the precise definition of a 'large' exchange economy.

Let $\mathscr{P}$ denote the set of preference relations $\precsim$ on R^l_+ (i.e., $\precsim$ is a reflexive, transitive complete and continuous binary relation) and $\mathscr{P}_{\mathrm{mo}}$ is the set of monotonic preference relations (i.e., $x \leqq y, x \neq y$ implies $x \prec y$). We endow the set $\mathscr{P}_{\mathrm{mo}}$ with a metric, the metric of closed-convergence [for details, see Kannai (1970) and Hildenbrand (1970 or 1974)]. A distribution on $\mathscr{P}_{\mathrm{mo}} \times R^l_+$ then means a probability measure on the σ-algebra $\mathscr{B}(\mathscr{P}_{\mathrm{mo}} \times R^l_+)$ of Borel sets in $\mathscr{P}_{\mathrm{mo}} \times R^l_+$.

An *atomless economy* $\mathscr{E}$ is defined as a measurable mapping $\mathscr{E}$ of an atomless[1] measure space $(A, \mathscr{A}, \nu)$ into $\mathscr{P}_{\mathrm{mo}} \times R^l_+$ such that the mean endowment vector

[1]Measure space always means $\nu(A) = 1$ and $\nu(B) \geqq 0$ for even $B \in \mathscr{A}$; a measure space $(A, \mathscr{A}, \nu)$ is called *atomless* if for every $E \in \mathscr{A}$ with $\nu(E) > 0$ there is $S \in \mathscr{A}$ with $0 < \nu(S) < \nu(E)$.

$\int_A e \circ \mathscr{E}\,\mathrm{d}\nu$ exists and is strictly positive (e denotes the projection of $\mathscr{P}_{\mathrm{mo}} \times R^l_+$ onto R^l_+).

The *distribution* $\mu_{\mathscr{E}}$ of an atomless economy $\mathscr{E}$ is the image measure of ν with respect to the mapping $\mathscr{E}$, i.e.,

$$\mu_{\mathscr{E}}(B) = \nu(\{a \in A | \mathscr{E}(a) \in B\})$$

for every Borel set in $\mathscr{P}_{\mathrm{mo}} \times R^l_+$.

An integrable function f of A into R^l_+ is called a *Walras allocation* for economy $\mathscr{E}$ (or a competitive equilibrium) if there is a price vector $p \in R^l$, $p \neq 0$, such that

(i) a.e. in A, $f(a) \in \varphi(\mathscr{E}(a), p)$, i.e., the vector $f(a)$ is a maximal element for $\precsim_{\mathscr{E}(a)}$ in the budget-set $\{x \in R^l_+ | p \cdot x \leqq p \cdot e(\mathscr{E}(a))\}$;

(ii) $\int f\,\mathrm{d}\nu = \int e \circ \mathscr{E}\,\mathrm{d}\nu$, i.e., mean demand equals mean supply.

The set of all Walras allocations of the economy $\mathscr{E}$ is denoted by $W(\mathscr{E})$. The corresponding set of equilibrium prices is denoted by $\Pi(\mathscr{E})$.

It has been shown by Aumann (1966) that for every atomless economy

$$\mathscr{E} : (A, \mathscr{A}, \nu) \to \mathscr{P}_{\mathrm{mo}} \times R^l_+$$

there exists a Walras allocation. Thus the sets $W(\mathscr{E})$ and $\Pi(\mathscr{E})$ are non-empty.

3. Equilibrium prices and distributions

If we consider only the set $\Pi(\mathscr{E})$ of equilibrium *prices* of the economy $\mathscr{E}$ – not the corresponding Walras allocations $W(\mathscr{E})$ – then the situation is quite easy:

Proposition. Let $\mathscr{E}_i : (A_i, \mathscr{A}_i, \nu_i) \to \mathscr{P}_{\mathrm{mo}} \times R^l_+$ $(i = 1, 2)$ *be two atomless economies with the same distribution* $\mu_{\mathscr{E}_1} = \mu_{\mathscr{E}_2}$. *Then*

$$\Pi(\mathscr{E}_1) = \Pi(\mathscr{E}_2).$$

Proof. A price vector p belongs to $\Pi(\mathscr{E}_i)$ if and only if $\int e \circ \mathscr{E}_i\,\mathrm{d}\nu_i \in \int \varphi(\mathscr{E}_i(\cdot), p)\,\mathrm{d}\nu_i$, i.e., the mean supply vector belongs to the mean demand set associated with the price vector p. By the change-of-variable formula we have

$$\int_{A_i} e \circ \mathscr{E}_i\,\mathrm{d}\nu_i = \int_{\mathscr{P}_{\mathrm{mo}} \times R^l_+} e\,\mathrm{d}\mu_{\mathscr{E}_i}.$$

But also the mean demand set $\int \varphi(\mathscr{E}_i(\cdot), p)\,\mathrm{d}\nu_i$ depends only on the distribution $\mu_{\mathscr{E}_i}$ of $\mathscr{E}_i$. Indeed one has

$$\int_A \varphi(\mathscr{E}_i(\cdot), p)\,\mathrm{d}\nu_i = \int_{\mathscr{P}_{\mathrm{mo}} \times R^l_+} \varphi(\cdot, p)\,\mathrm{d}\mu_{\mathscr{E}_i}$$

since the measure space $(A_i, \mathscr{A}_i, \nu_i)$ is atomless [Hildenbrand (1974, chapter 1, proposition 4)]. Thus the condition for p to be an equilibrium price depends only on the distribution of $\mathscr{E}$, and this clearly implies the proposition. Q.E.D.

4. Walras allocations and distributions

We now want to go a step further and we shall consider the equilibrium allocation as well. An allocation was defined as a mapping f of the set A of agents into the commodity space, i.e., an assignment of commodities to individual agents. Now, if we consider the distribution $\mu_{\mathscr{E}}$ of an economy $\mathscr{E}$ as the basic concept, while we regard the set of agents as not being intrinsic, we have to follow the same change of view for allocations, that is to say, we consider the distribution $\mathscr{D}f$ of the allocation f in the commodity space. Then we can no longer speak of a particular commodity bundle allocated to a particular individual agent. However, given a certain subset S in the commodity space, we can speak of the *fraction* of the totality of agents having their allocation in this set.

Unlike the case of equilibrium prices, the set $\mathscr{D}W(\mathscr{E})$ of distributions of all Walras allocations $W(\mathscr{E})$ is not completely determined by the distribution $\mu_{\mathscr{E}}$ of agents' characteristics. The following example is discussed in detail in Kannai (1970).

Example. We shall define two economies $\mathscr{E}_1$ and $\mathscr{E}_2$ with the same distribution $\mu_{\mathscr{E}_1} = \mu_{\mathscr{E}_2}$. The space of agents for both economies is the unit interval [0, 1] with Lebesgue measure λ. All agents, in both economies, have the same preference relation whose indifference curves are pictured in the following figure:

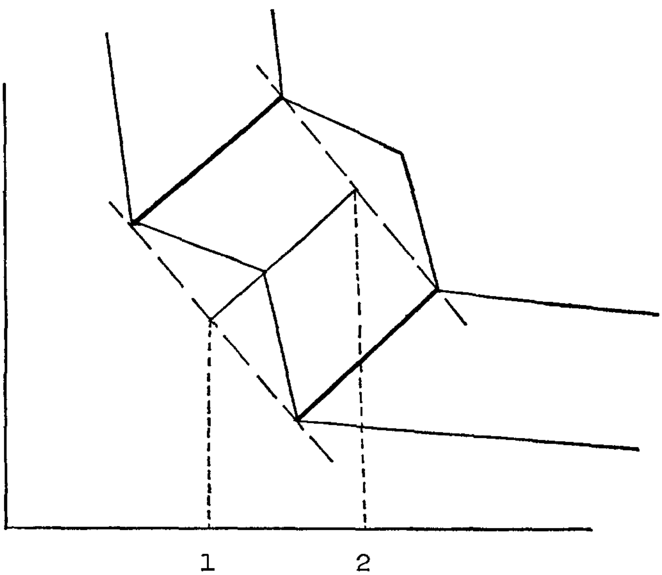

Fig. 1

The endowments are given by

$$e(\mathscr{E}_1(t)) = (1+t, 1+t), \quad t \in [0, 1],$$

and

$$e(\mathscr{E}_2(t)) = \begin{cases} (1+2t, 1+2t) & \text{if } t \in [0, \frac{1}{2}), \\ (2t, 2t) & \text{if } t \in [\frac{1}{2}, 1]. \end{cases}$$

One easily verifies that for the second economy $\mathscr{E}_2$ there is a Walras allocation f such that its distribution $\mathscr{D}f$ is the uniform distribution on the darkened two segments in the above figure. However, this distribution cannot be obtained as a distribution of a Walras allocation in the first economy $\mathscr{E}_1$, hence

$$\mathscr{D}W(\mathscr{E}_1) \neq \mathscr{D}W(\mathscr{E}_2).$$

Theorem 1. *Let $\mathscr{E}_1$ and $\mathscr{E}_2$ be two atomless economies with the same distribution $\mu_{\mathscr{E}_1} = \mu_{\mathscr{E}_2}$ on $\mathscr{P}_{\text{mo}} \times R^l_+$. Then the sets $\mathscr{D}W(\mathscr{E}_1)$ and $\mathscr{D}W(\mathscr{E}_2)$ of distributions of Walras allocations of $\mathscr{E}_1$ and $\mathscr{E}_2$, respectively, have the same closure with respect to the weak convergence*;

$$\overline{\mathscr{D}W(\mathscr{E}_1)} = \overline{\mathscr{D}W(\mathscr{E}_2)}.$$

Proof. By the above Proposition we have $\Pi(\mathscr{E}_1) = \Pi(\mathscr{E}_2)$. Let $p \in \Pi(\mathscr{E}_i)$, and denote by $W(\mathscr{E}_i, p)$ the set of Walras allocations for the economy $\mathscr{E}_i$ which are associated with the equilibrium price p $(i = 1, 2)$. Thus

$$W(\mathscr{E}_i, p) = \{f \in \mathscr{L}_{\varphi_i} \mid \int_{A_i} f \, \mathrm{d}\nu_i = \int e \circ \mathscr{E}_i \, \mathrm{d}\nu_i\},$$

where $\mathscr{L}_{\varphi_i}$ denotes the set of all integrable selections of the correspondence φ_i of $(A_i, \mathscr{A}_i, \nu_i)$ into R^l; $a \mapsto \varphi(\mathscr{E}_i(a), p)$ $(i = 1, 2)$.

In order to prove Theorem 1 it suffices to show that

$$\overline{\mathscr{D}W(\mathscr{E}_1, p)} = \overline{\mathscr{D}W(\mathscr{E}_2, p)}.$$

But this follows immediately from Theorem 3 in Hart–Kohlberg (1974). Indeed, the two correspondences φ_1 and φ_2 are integrably bounded since $p \gg 0$ and since $e \circ \mathscr{E}_i$ is integrable by assumption. Clearly φ_i is closed-valued. It is not hard to show that φ_i has a measurable graph [e.g., use D.II.3., Proposition 1.(b), and chapter 1.2., proposition 2 in Hildenbrand (1974)]. Finally, φ_1 and φ_2 are equally distributed. Indeed, for every Borel set S in R^l we have

$$\nu_1(\varphi_1^{-1}(S)) = \nu_1(\mathscr{E}_1^{-1}\{(\precsim, e) | \varphi(\precsim, e, p) \cap S \neq \emptyset\}) =$$
$$\nu_2(\mathscr{E}_2^{-1}\{(\precsim, e) | \varphi(\precsim, e, p) \cap S \neq \emptyset\}) = \nu_2(\varphi_2^{-1}(S)),$$

since, by assumption, $\mathscr{E}_1$ and $\mathscr{E}_2$ are equally distributed. Hence all assumptions of Theorem 3 in Hart–Kohlberg (1974) are fulfilled. This completes the proof of Theorem 1. Q.E.D.

5. Standard representation

The natural question to ask now is whether there are economies for which the set $\mathscr{D}W(\mathscr{E})$ is closed. The example given above shows that it does not suffice that there are many agents, i.e., that the economy is atomless. What one needs actually is that there are many agents of every characteristic in support of the measure μ.

Let μ be a distribution of agents' characteristics. Then we define an economy $\mathscr{E}^{\mu}$ with distribution μ, which we shall call the *standard representation of* μ; the atomless measure space of agents is given by $A = (\mathscr{P}_{\mathrm{mo}} \times R^l_+) \times [0, 1]$ with the product measure $\nu = \mu \otimes \lambda$, where λ is the Lebesgue measure on [0, 1]. The mapping $\mathscr{E}^{\mu}$ is simply the projection, i.e., $\mathscr{E}^{\mu}(\precsim, e, \xi) = (\precsim, e)$ for every $(\precsim, e, \xi) \in \mathscr{P}_{\mathrm{mo}} \times R^l_+ \times [0, 1]$.

With this notation we can state the following result:

Theorem 2. *Let* μ *be a measure on* $\mathscr{P}_{\mathrm{mo}} \times R^l_+$ *with* $\int e \,\mathrm{d}\mu \gg 0$. *Then* $\mathscr{D}W(\mathscr{E}^{\mu})$ *is closed.*

Proof. We give a proof under the additional assumption that the measure μ is concentrated on a compact set. The reader familiar with the concept of tightness will easily see that this assumption is not needed. Let $f_n \in W(\mathscr{E}^{\mu})$ and assume that the sequence $(\mathscr{D}f_n)$ of distributions converges say $\mathscr{D}f_n \to \delta$. We have to show that there exists $f \in W(\mathscr{E}^{\mu})$ with distribution $\mathscr{D}f = \delta$. Consider the joint distribution τ_n of the mapping $(\mathscr{E}^{\mu}, f_n)$ $(n = 1, \ldots)$. All mappings $(\mathscr{E}^{\mu}, f_n)$ take values in a compact subset of $(\mathscr{P}_{\mathrm{mo}} \times R^l) \times R^l$. Indeed, since the set $\Pi(\mathscr{E}^{\mu})$ is closed and strictly positive and since by assumption all endowment vectors belong to a compact set, it follows that the sequence (f_n) is bounded. It is well known that the set of probability measures on a compact metric space is weakly compact. Therefore we can assume without loss of generality that the sequence (τ_n) is weakly convergent, say $\tau_n \to \tau$. Note that the marginal distributions of τ on $\mathscr{P}_{\mathrm{mo}} \times R^l_+$ and R^l_+ are μ and δ, respectively.

We now apply Skorokhod's Theorem [see e.g., Hildenbrand (1974, D.1)] to the sequence $\tau_n \to \tau$. Thus there is a measure space $(A, \mathscr{A}, \nu)$ and measurable mappings $(\tilde{\mathscr{E}}_n, \tilde{f}_n)$ $(n = 1, 2, \ldots)$ and $(\tilde{\mathscr{E}}, \tilde{f})$ of A into $(\mathscr{P}_{\mathrm{mo}} \times R^l_+) \times R^l_+$ such that $(\tilde{\mathscr{E}}_n, \tilde{f}_n) \to (\tilde{\mathscr{E}}, \tilde{f})$ a.e. in A, and the distributions of $(\tilde{\mathscr{E}}_n, \tilde{f}_n)$ and $(\tilde{\mathscr{E}}, \tilde{f})$ are τ_n and τ, respectively. Since $(\mathscr{E}^{\mu}, f_n)$ and $(\tilde{\mathscr{E}}_n, \tilde{f}_n)$ have the same joint distribution τ_n, and since $f_n \in W(\mathscr{E}^{\mu})$, it follows that $\tilde{f}_n \in W(\tilde{\mathscr{E}}_n)$ $(n = 1, \ldots)$. But this, together with the pointwise convergence of the sequence $(\tilde{\mathscr{E}}_n, \tilde{f}_n)$ to $(\tilde{\mathscr{E}}, \tilde{f})$ implies by standard arguments that $\tilde{f} \in W(\tilde{\mathscr{E}})$. Therefore, in order to complete the proof we have to show that there is an allocation f for the economy $\mathscr{E}^{\mu}$ such that the joint distribution of $(\mathscr{E}^{\mu}, f)$ is equal to τ. Indeed, it then follows that $f \in W(\mathscr{E}^{\mu})$ and $\mathscr{D}f = \delta$ since $\tilde{f} \in W(\tilde{\mathscr{E}})$, and the joint distribution of $(\tilde{\mathscr{E}}, \tilde{f})$ is equal to τ. Recall that the marginal distributions of τ are μ and δ, respectively.

Thus we have to construct[2] a measurable mapping of the standard representation $(\mathscr{P}_{mo} \times R^l_+) \times [0, 1]$ into $(\mathscr{P}_{mo} \times R^l_+) \times R^l_+$ whose distribution is τ and which is of the special form:

$$(t, \xi) \mapsto (t, f(t, \xi))$$

for every $t \in \mathscr{P}_{mo} \times R^l_+$ and $\xi \in [0, 1]$.

This mapping is obtained in the following way. Given the measure τ on the product $(\mathscr{P}_{mo} \times R^l_+) \times R^l_+$, let δ_t denote the (regular) conditional probability distribution for $(t, \xi) \mapsto \xi$ given t; thus δ_t is a probability measure on R^l_+, the mapping $t \mapsto \delta_t(V)$ is measurable for every Borel set V in R^l_+ and one has

$$\tau(U \times V) = \int_U \delta_t(V)\, \mu(\mathrm{d}t)$$

for Borel sets U in $\mathscr{P}_{mo} \times R^l_+$ and V in R^l_+ [see e.g., Loève (1963, sections 26 and 27)].

Every measure δ_t on R^l_+ can be obtained as the image of the Lebesgue measure of a mapping $f(t, \cdot)\colon [0, 1] \to R^l_+$. Then we define the mapping $(t, \xi) \mapsto (t, f(t, \xi))$ which clearly has the distribution τ. The functions $f(t, \cdot)$ can be chosen such that the function $f(\cdot, \cdot)$ is measurable. Indeed, by the well-known Isomorphism Theorem we can assume without loss of generality that the measures δ_t are defined on [0, 1]. Consider then the (accumulative) distribution function $F(t, \cdot)$ of δ_t, i.e., $F(t, \xi) = \delta_t[0, \xi]$. Thus $F(t, \cdot)$ is a right-continuous function of [0, 1] into [0, 1] for every t, and $F(\cdot, \xi)$ is measurable for every ξ. This implies (see the lemma, below) that the function $F(\cdot, \cdot)$ is measurable. Hence, by choosing the function $f(t, \cdot)$ to be the reflexion on the diagonal of $F(t, \cdot)$, i.e.,

$$f(t, \xi) = \begin{cases} 0 & \text{if } \xi < F(t, 0), \\ \sup \{\eta | F(t, \eta) \leqq \xi\} & \text{otherwise,} \end{cases}$$

one obtains a function $f(\cdot, \cdot)$, which is measurable. Q.E.D.

Lemma. Let $(T, \mathscr{J})$ be a measurable space and f a function of $T \times [0, 1]$ into $[0, 1]$ such that

(a) *$f(\cdot, \xi)$ is $\mathscr{J}$-measurable for every $\xi \in [0, 1]$;*

(b) *$f(t, \cdot)$ is right continuous for every $t \in T$.*

Then the function f is $\mathscr{J} \otimes \mathscr{B}[0, 1]$-measurable.

Proof. For every integer $n = 1, \ldots$ we define the function f_n of $T \times [0, 1]$ into [0, 1] by

$$f_n(t, \xi) = f\left(t, \frac{k}{n}\right), \quad \text{if} \quad \frac{k-1}{n} \leq \xi < \frac{k}{n} \quad (k = 1, \ldots, n)$$

[2]We learned from David Freedman that constructions of this type belong to the folklore in Probability Theory. Since we could find no reference we thought it might be helpful to some readers if we outline the construction.

$f_n(t, 1) = f(t, 1).$

Clearly, by assumption (a) the function f_n restricted to $T\times[(k-1)/n, k/n]$ $(k = 1, \ldots, n)$ is measurable, and hence f_n is measurable. By assumption (b) we have $\lim_n f_n(t, \xi) = f(t, \xi)$ for every (t, ξ). Hence f is measurable. Q.E.D.

6. Upper hemicontinuity of Walras allocations in distribution

For every distribution μ of agents' characteristics we define

$$\mathscr{D}W(\mu) = \mathscr{D}W(\mathscr{E}^{\mu}).$$

Of course, we could have chosen any atomless economy $\mathscr{E}$ with distribution μ and define

$$\mathscr{D}W(\mu) = \overline{\mathscr{D}W(\mathscr{E})}.$$

Theorem 3. *Let T be a compact subset in $\mathscr{P}_{\mathrm{mo}}\times R^l_+$. Then the correspondence $\mu \mapsto \mathscr{D}W(\mu)$ defined on the set of distribution on T is compact-valued and u.h.c. at every distribution μ with strictly positive mean endowments $\int e \, \mathrm{d}\mu \gg 0$.*

Proof. First we remark that $\mathscr{D}W(\mu) \neq \emptyset$. Let the sequence (μ_n) converge to μ and let $\delta_n \in \mathscr{D}W(\mu_n)$. We have to show that there is a weakly converging subsequence (δ_{n_q}) whose limit δ belongs to $\mathscr{D}W(\mu)$. Since $\delta_n \in \mathscr{D}W(\mu_n)$ there exist a Walras allocation f_n and a corresponding equilibrium price p_n for the standard representation $\mathscr{E}^{\mu_n}$ such that $\mathscr{D}f_n = \delta_n$. It is not difficult to show [or, see Hildenbrand (1974, chapter 2, proposition 6)] that every limit point p of the sequence (p_n) is strictly positive. This implies that the sequence (f_n) is bounded. Now consider the joint distribution τ_n of the mapping $(\mathscr{E}^{\mu_n}, f_n)$. The sequence (τ_n) is contained in a compact set, and, therefore, we can assume without loss of generality that the sequence (τ_n) is convergent, say $\tau_n \to \tau$. Using Skorokhod's Theorem as in the proof of Theorem 2 one shows that there is an atomless economy $\mathscr{E}$ and a Walras allocation $f \in W(\mathscr{E})$ such that the distribution of $(\mathscr{E}, f)$ is equal to τ. Since the marginal distributions of τ are μ and δ, respectively, it follows that $\delta \in \mathscr{D}W(\mathscr{E})$, and hence $\delta \in \mathscr{D}W(\mu)$. Q.E.D.

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