

Reply to Binmore and Samuelson

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I heartily agree with many of the points made by Binmore and Samuelson. First, that the evolutionary approach to game theory is very important, though perhaps not to the exclusion of all other approaches. Second, that in perfect information (PI) games, backward induction (BI) may yield choices that are not only unwise and unreasonable, but quite simply irrational. Third, that interpretational issues are very important in these matters, and that in discussing them, one cannot rely exclusively on mathematics. Fourth, that in game theory and decision theory in general, and in the matter at hand in particular, one cannot restrict oneself to material implication, but must use what Binmore and Samuelson call 'subjunctive conditionals'. Fifth, that deleting weakly dominated strategies is not 'harmless'. Sixth, that in considering equilibria, one should not restrict attention to those that are 'trembling hand' perfect. Seventh, that the experimental results in the Ultimatum Game constitute an important phenomenon that cannot and should not be ignored or dismissed, and on which game theory has much of importance to say. And I could go on, listing many more points of agreement with their positions.

These positions are argued by Binmore and Samuelson at great length, with fervour and flair and even some reasoned arguments. I have promoted many of them myself – some in the very paper they are attacking (Aumann, 1995) and others previously (Aumann, 1985, 1987, 1992a, 1992b). Indeed, the point about 'subjunctive conditionals' is absolutely central to my position (see Aumann, 1995, section 4b). I am delighted to have Binmore and Samuelson on my side in this matter, as well as in the others listed above.

The one cardinal point on which I disagree with them is my contention that common knowledge of rationality (CKR) implies backward induction. Wisely, they have made their position on this impregnable. Their arguments cannot be refuted, as they make no arguments. To be sure, they do *assert* in one place that CKR does not imply BI, and in another that there are flaws in my reasoning. But nowhere do they *show* that CKR does not imply BI, or even attempt a reasoned argument to that effect; and nowhere do they point to a single flaw, either logical or conceptual, in *my* argument.

To challenge the result that CKR implies BI, one must either find a logical flaw in my demonstration, or challenge my formulations of the underlying concepts – rationality and knowledge. Binmore and Samuelson do neither. Instead, they devote all their energy to arguing that in certain PI games, the BI choices may be (or are) unreasonable. That is correct; but it does not preclude CKR implying BI. You cannot discredit '*p* implies *q*' by questioning *q*; perhaps *p* is also questionable. People who chide others – wrongly! – for confusing subjunctive conditionals with material implications should not themselves fall into the much more elementary error of confusing an implication with its consequent. Physician, heal thyself!

The fact of the matter is that there is a big difference between rationality and CKR, a point that Binmore and Samuelson consistently slur over. CKR is a very

strong assumption, much stronger than simple rationality, and in some situations may just be too much to ask for (Aumann, 1992b). It is this, and this only, that accounts for the strangeness of the BI outcome in some PI games.

There is nothing more to say; I cannot defend myself against an attack that has no substance – is all smoke and no fire. A more detailed discussion of the issues may be found in Aumann (1995). As for the question in Binmore and Samuelson's title – 'Rationalizing Backward Induction?' – my answer is 'no; showing that CKR implies BI'.

References

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