



## CORRESPONDENCE

Editors,

A recent paper of Binmore appears to contain a fundamental logical error.

At the beginning of his contribution to the recent symposium on information and knowledge, Binmore ([Why the Distinction Between Knowledge and Belief Might Matter EJW April 2005](#)) adduces the following motivating example:

Alice is a perfectly rational decision-maker who values her own safety. She therefore won't step in front of a car when crossing the road. I am so sure of my facts that I attribute probability one to this assertion. But what was my reasoning process in coming to this conclusion? I have to contemplate Alice comparing the consequences of stepping in front of a car with staying on the kerb. But how can Alice or I evaluate the implications of the former event, which we know is impossible? In mathematical logic, anything whatever can be deduced from a contradiction.

The entire seven-page article ensues in this spirit.

With all our respect and admiration for Ken Binmore, we are dumbfounded by his analysis. The assertion in question is, "Alice won't step in front of a car when crossing the road." Let's call this  $p$ . Binmore became convinced of  $p$  by a reasoning process involving several elements, including Alice's concern for her safety. He then asks, "What was my reasoning in coming to this conclusion?" That is, he wishes to review the reasoning leading to  $p$ . To do so he—and Alice—contemplate the consequences of  $\neg p$  (the negation of  $p$ ); namely, that she does step in front of a car. Considering the consequences of  $\neg p$  in seeking to establish  $p$  is a universally accepted

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procedure both in everyday thought and in formal logic; it is called reasoning “per absurdum,” or “indirectly.” So far, so good.

Now Binmore asks, “How can Alice or I evaluate the implications of the former event [i.e.  $\neg p$ ], which we know is impossible? In mathematical logic, anything whatever can be deduced from a contradiction.” But Ken, that you know  $\neg p$  to be impossible is only because you previously convinced yourself of  $p$ . You are now reviewing the reasoning leading to that previous conclusion, i.e., to  $p$ . In this review, surely you cannot assume  $p$  itself—you cannot assume what you wish to prove!

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