48 Disadvantageous Monopolies

1. Introduction

It seems intuitively obvious that in a monopolistic market, the monopolist has an advantage because he can avoid competition, i.e., force an outcome that is better for himself than one that would be a result of competition. A formulation of this principle in terms of the core is the following:

CONJECTURE. In a monopolistic market, for each core allocation $\mathbf{x}$ there is a competitive allocation $\mathbf{y}$ whose utility to the monopolist[1] is $\leqslant$ that of $\mathbf{x}$.

This conjecture appears[2] in [3], where it is stated in terms of a market in which the traders form a measure space with a single atom (the "monopolist") and a nonatomic part. It is proved in [3] that the conjecture is correct in the case of homogeneous markets, i.e., markets in which all traders have the same utility function, and this utility function is homogeneous. In that case the core is generally quite large, whereas there is a unique competitive allocation; the latter is, from the point of view of the monopolist, the worst allocation in the core.

In this note the above conjecture is settled, in the negative. We bring three examples. In Example A, the core is quite large, there is a unique competitive allocation, and from the monopolist's viewpoint, the competitive allocation is approximately in the middle of the core. Example B is a variant of Example A. In it, the core is again quite large, there is a unique competitive allocation, and from the monopolist's viewpoint,

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[1] That is, the utility of the bundle allocated to the monopolist under the allocation in question.

[2] Albeit not as a conjecture, but as an open problem.

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the competitive allocation is the *best* in the core. Thus the monopoly is a disadvantage; the monopolist would do well to "go competitive", i.e., split himself into many competing small traders.[3] In Example C, the core consists of exactly two points, one competitive and one not; and the competitive point is the better of the two from the monopolist's viewpoint. Thus again, the monopolist would do well to "go competitive."

The examples are formulated in terms of "mixed markets," i.e., the traders form a measure space consisting of a single atom $\{a\}$ and a non-atomic part, which will be called the "ocean." All examples are 2-commodity markets, with all of one commodity initially concentrated in the hands of the atom and all of the other initially held by the ocean. Thus a is a "monopolist" both in the sense of being an atom and in the sense that he initially holds a "corner" on one of the two commodities.

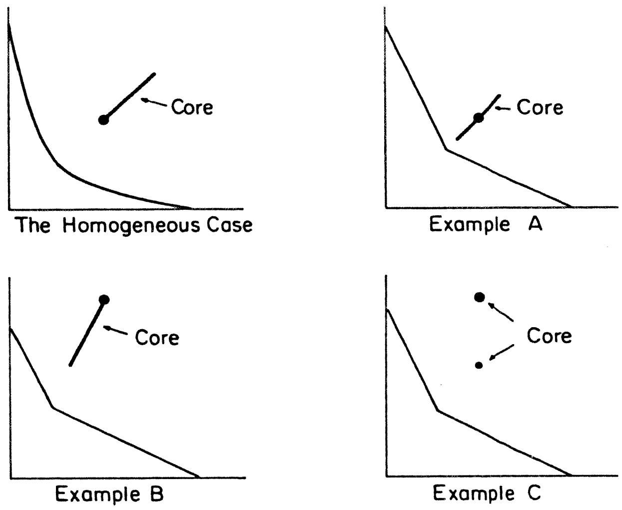

FIGURE 1

The chief features of the examples are illustrated in Fig. 1. This figure is drawn from the viewpoint of the atom only. In each case, we indicate the core (or more precisely, the monopolist's bundles in the core); the position of the competitive allocation is indicated by a heavy dot. The indifference level corresponding to the "personal minimum" of the monopolist (i.e. the utility of his initial bundle) is also shown.

The examples are presented in Section 2. Section 3 is devoted to a discussion of the implications of these examples, and the relation of the core analysis with that by means of other tools.

[3] In which case the core would consist of the competitive allocation only; see [1].

2. The Examples

It is assumed that the reader is familiar with the terminology and notation of Core theory, in particular as applied to mixed markets; see [3], for example. The atom and the ocean are denoted $\{a\}$ and T_0 respectively; each has measure 1. In all the examples, the initial bundle density of trader t is given by

$$\begin{aligned} \mathbf{i}(t) &= (0, 1) \quad \text{if} \quad t \in T_0\,; \\ &= (1, 0) \quad \text{if} \quad t = a. \end{aligned} \tag{2.1}$$

To define the preference relations, we express the ocean as the union of two disjoint sets U and V of measure $\frac{1}{2}$ each; traders in U will have different preference relations from those in V.

To simplify the notation, we write $x = (\alpha, \beta)$ rather than $x = (x^1, x^2)$ for the bundle densities.

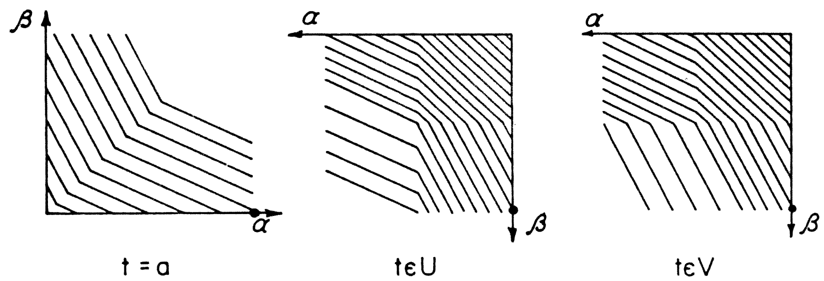

Fig. 2. Indifference maps in Example A.

Example A. Let

$$h(\alpha, \beta) = \min(2\alpha + \beta, 2(\alpha + \beta) - 1/2, \alpha + 2\beta, (\alpha + 2\beta)/2 + 3/4).$$

Define the utility function u_t of trader t by

$$u_t(x) = u_t(\alpha, \beta)$$
$$= \begin{cases} \alpha + \beta + \min(\alpha, \beta) = \min(\beta + 2\alpha, \alpha + 2\beta), & \text{for} \quad t = a; \\ h(\alpha, \beta), & \text{for} \quad t \in U; \\ h(\beta, \alpha), & \text{for} \quad t \in V. \end{cases}$$

The indifference maps and initial bundle densities are illustrated in Fig. 2; the maps for $t \in U$ and $t \in V$ have been inverted, so as to prepare the way for the use of the Edgeworth box below.

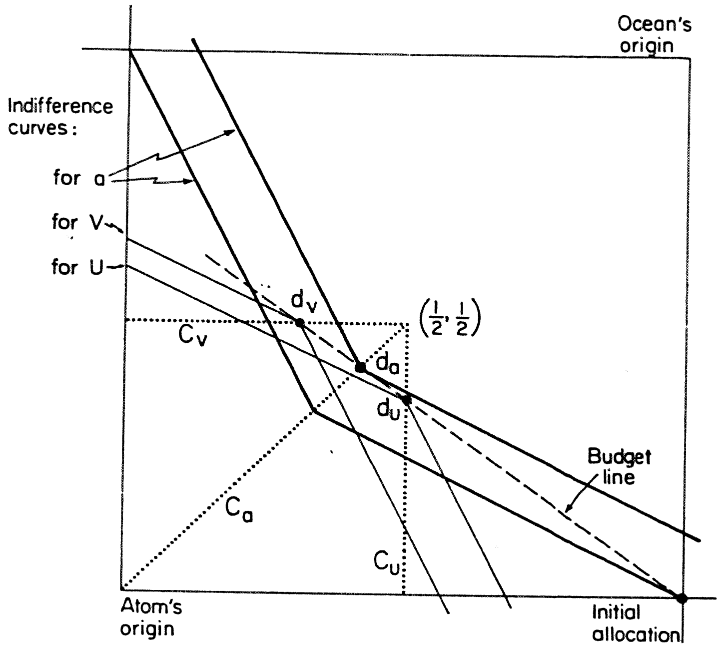

FIGURE 3

There is only one competitive allocation in this market, given by $\mathbf{x}(t) = (\frac{1}{2}, \frac{1}{2})$ for all t; the price vector is $p = (1, 1)$. It is easily verified that this is indeed a competitive equilibrium. To see that there is no other, let $p = (p^1, p^2)$ be a price vector with $1 > p^1/p^2 > \frac{1}{2}$, and refer to the Edgeworth Box in Fig. 3. The dotted lines C_a, C_U, and C_V are called *critical lines* for $t = a$, $t \in U$, and $t \in V$ respectively; on these lines the respective utility functions are not differentiable. The demand of each trader t is determined to be the intersection of the budget line with his critical line. Since $\mu(V) = \mu(U) = \frac{1}{2}$, the ocean's total demand is the midpoint of the segment connecting the demands d_U and d_V of traders in U and V respectively; since the demand d_a of the atom does not coincide[4] with this midpoint, total demand does not match total supply.

Next, we turn to the core. The atom's bundles in the core consists of the line segment connecting (3/8, 3/8) to (5/8, 5/8) (Fig. 1). At the top endpoint the oceanic players get their personal minimum, and so the atom cannot hope to get more. The bottom endpoint is, however, more than the atom's personal minimum (1/3, 1/3). The unique competitive allocation sits squarely in the middle of the core. Since our interest lies chiefly in the points of the core below the competitive allocation, we shall concen-

[4] One must recall that in the Edgeworth box, the atom and the oceanic traders have different origins. Thus if in the box, d_a coincides with the point half-way between d_U and d_V, then algebraically, $d_a + (d_U + d_V)/2 = (1, 1)$ (*not* $d_a = (d_U + d_V)/2$).

trate on them; the reader may verify for himself those of our other assertions that he wishes.

We make use of Theorem A* (Theorem 5.2) of [3]; applied to a mixed market such as ours[5], this yields that an individually rational allocation $\mathbf{x}$ is in the core if and only if there is a price vector p such that

$$\textit{for almost all } t,\ \mathbf{x}(t) \textit{ is maximal in } \{x \in \Omega : p \cdot x \leqslant p \cdot \mathbf{x}(t)\}, \text{ and} \tag{2.2}$$

$$\textit{for almost all } t \textit{ in the ocean, } p \cdot \mathbf{i}(t) \geqslant p \cdot \mathbf{x}(t). \tag{2.3}$$

If p satisfies (2.2), then it is called a vector of *efficiency prices*; the line $\{x: p \cdot x = p \cdot \mathbf{x}(t)\}$ is called the *efficiency budget line.*

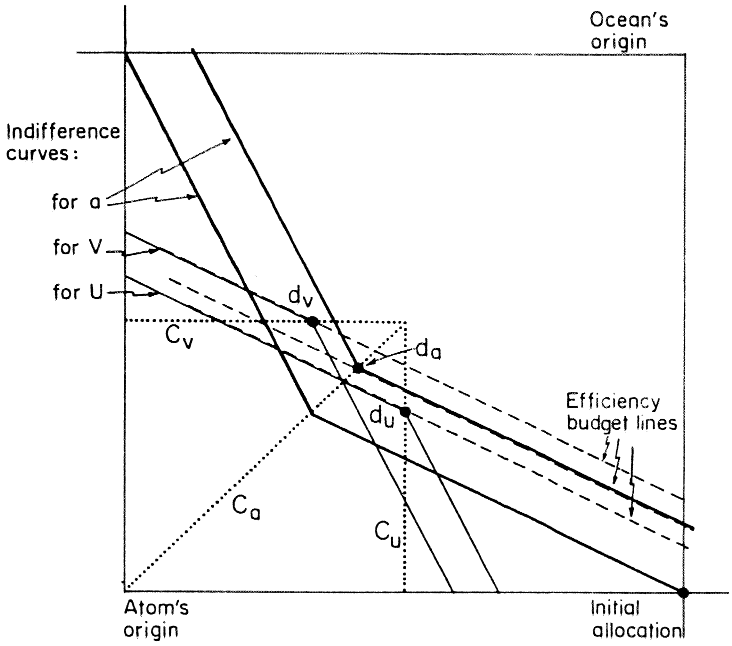

FIGURE 4

Refer to Fig. 4. For a given bundle d_a of the atom between (3/8, 3/8) and (1/2, 1/2), choose d_U and d_V to lie on C_U and C_V respectively, so that d_a is exactly half-way between them. The efficiency budget lines with slope $-\frac{1}{2}$ through d_a, d_U, and d_V respectively (dashed in the figure) will support the indifference curves and pass above the initial allocation; this means that (2.2) and (2.3) are satisfied, and so we get a core allocation. When d_a is under (3/8, 3/8), any efficiency budget line for d_U will pass under the initial allocation, so that (2.3) is violated.

[5] That is, a market with precisely one atom, in which the ocean cannot gain by trading within itself only.

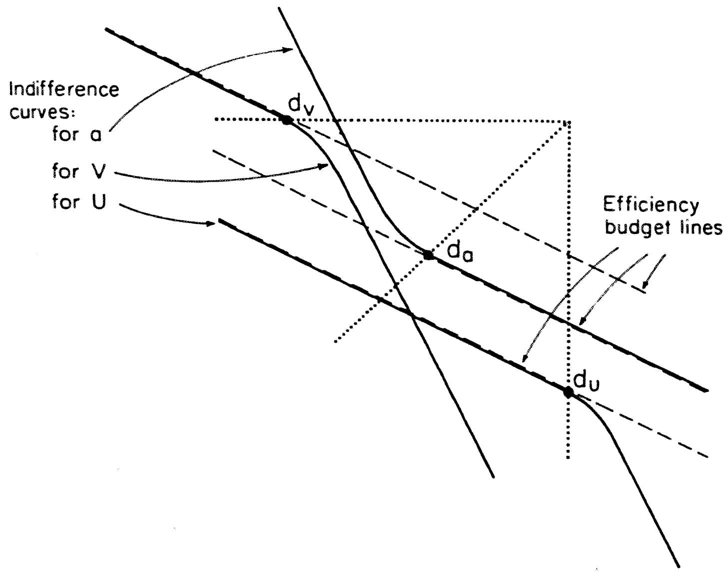

FIG. 5. Smoothing in Example A.

A differentiable version of this example is constructed by appropriate smoothing in the neighborhood of the "corners." The smoothing must be carried out so that the efficiency budget lines continue to support the indifference curves at the points d_a, d_U, and d_V respectively (Fig. 5). The above argument, proving that this yields a core allocation, then remains unchanged. Moreover, if the new indifference curves are sufficiently close to the old ones, we cannot get new competitive allocations at any appreciable distance from (1/2, 1/2). Thus although it is conceivable that some competitive allocations are added, we will certainly still have a considerable part of the core yielding the atom less than the "worst" competitive allocation.

The example is also easily modified so that the utility functions are *strictly* quasiconcave (i.e., the preferences are strictly convex) as well as differentiable.

EXAMPLE B. The utility functions in this example are as follows:

$$u_t(x) = u_t(\alpha, \beta)$$
$$= \begin{cases} 4\alpha + 5\beta + 3\min(2\alpha, \beta) = \min(5(\beta + 2\alpha), 4(\alpha + 2\beta)), \\ \qquad \text{for} \quad t = a; \\ \alpha + 2\beta + 3\min(\alpha, 1/2) = \min(2(\beta + 2\alpha), \alpha + 2\beta + 3/2), \\ \qquad \text{for} \quad t \in U; \\ \beta + 2\alpha, \qquad \text{for} \quad t \in V. \end{cases}$$

The initial allocation is given by (2.1).

In this example the indifference curves are similar to those in the bottom half of the box in the previous example; the critical lines are, of course, different. For $t \in V$ there is no critical line; its place is taken by the α axis (see Fig. 6).

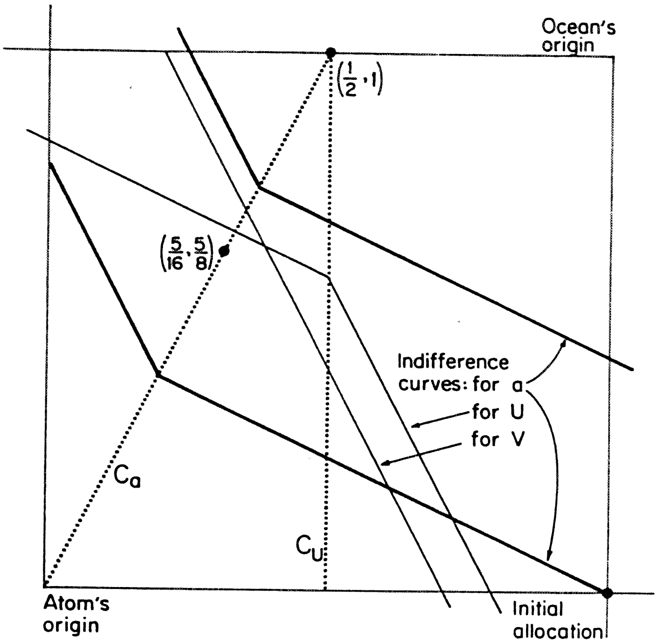

FIG. 6. Example B.

Arguments similar to those used in connection with Example A show that there is a unique competitive equilibrium, which yields the atom (1/2, 1); whereas in the core, the atom gets anything on the line connecting (5/16, 5/8) to (1/2, 1) (Fig. 6).

Like Example A, this example can also be "smoothed out," i.e., the utility functions can be made differentiable (and strictly quasiconcave). As in Example A, the smoothing may introduce new competitive allocations close to the original one. In any case, all but an arbitrarily small portion of the core will be worse for the monopolist than the worst competitive allocation.

EXAMPLE C. This example is perhaps the strangest of all. The utility functions for the oceanic players are defined as in Example B. The atom's utility function is given by

$$u_a(x) = u_a(\alpha, \beta) = \min(14(\beta + 2\alpha), 13(\alpha + 2\beta), 4(\alpha + 2\beta) + 18).$$

The initial allocation is given by (2.1). The critical lines as well as some indifference curves for the atom are given in Fig. 7.

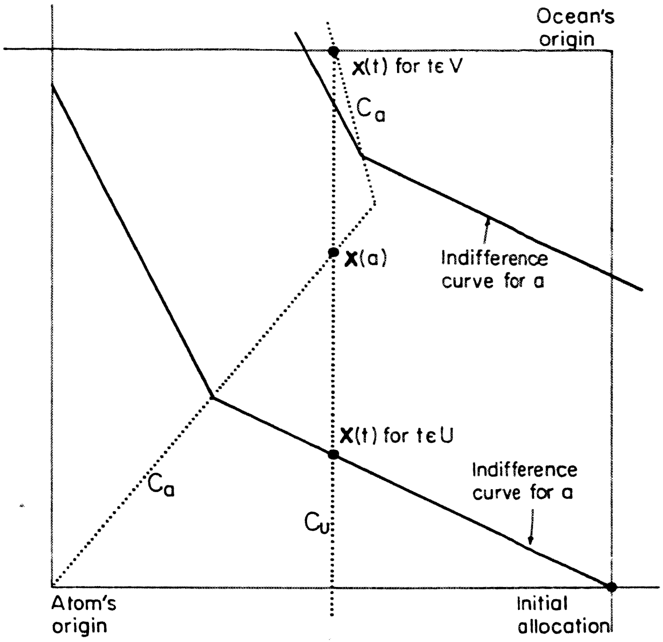

FIG. 7. Example C.

As before, there is a unique competitive allocation, which yields the atom (1/2, 1). There is exactly one other allocation **x** in the core; it is given by

$$\begin{aligned} \mathbf{x}(t) &= (1/2, 5/8) && \text{for } t = a \\ &= (1/2, 3/4) && \text{for } t \in U \\ &= (1/2, 0) && \text{for } t \in V. \end{aligned}$$

Like the previous examples, this example is easily smoothed, i.e. made differentiable (and strictly quasiconcave). Unlike in the previous examples, the smoothing leaves both the competitive allocation and the core entirely unchanged.

3. Discussion

Our first remark concerns the relation between Examples A and B. The reader might ask why we bring Example A at all, when Example B is both simpler (in the definition of the utilities) and more striking (in that the competitive allocation is *best* in the core for the monopolist). The answer is that Example B is in a certain sense atypical, because at the unique competitive allocation, the oceanic traders get their personal minima. Thus the atom cannot hope to get more than at the competitive allocation; if the core is to extend beyond the competitive allocation at all, it can only extend "below" it. Of course, it is still very surprising that the core does in fact extend below the competitive allocation, i.e., to get the best point in the core, the atom should dissolve. Nevertheless, had we brought this example alone, some readers might have dismissed it as atypical. Example A shows that the core may extend below the competitive allocation[6] even in a case that has no discernible atypical or extreme features.

Indeed, perhaps the most disturbing aspect of these examples is their utter lack of pathology. It would be difficult to ask for more sedate, well-behaved utility functions. They can be assumed differentiable to any degree and strictly quasiconcave, and the slopes of the indifference curves are bounded away from zero and bounded; in particular, therefore, any prices associated with these markets must be bounded away from zero and bounded. What makes the examples possible is that not all the small traders have the same preferences; this again, far from being a pathology, is the "usual" situation. One is almost forced to the conclusion that monopolies that are not particularly advantageous (like Example A) are probably the rule rather than the exception. This conclusion runs counter to common sense as well as to economic theory. Perhaps what is needed at this stage is a careful reappraisal of the ideas underlying the use of the core in economic analysis.

While we are not prepared to undertake such a reappraisal within the framework of this note, we can at least hint at one aspect of it. But first, let us see how these examples appear when looked at from the viewpoint of classical economic theory. According to the classical theory, the oceanic traders in a monopoly will act like price takers, i.e., they will maximize their utility given the prices set by the monopolist. The monopolist will set the prices so that the result of price-taking on the part of the ocean will maximize *his* utility. The kind of phenomenon illustrated for the core in this note is of course impossible in the classical theory. If the monopolist sets prices, he cannot end up worse off than at the competitive equilibrium, since he always has the option of setting the prices equal to competitive prices.[7] In our examples, the monopolist can choose p^1/p^2

[6] In cases of this kind, it must necessarily extend above as well. Indeed, the allocation at which the atom maximizes his utility subject to the oceanic traders getting their personal minima is in the core; this is because the oceanic traders initially hold only one good, and so cannot gain by trading among themselves only.

[7] Strictly speaking, this conclusion is justified only when the preferences of the oceanic traders are *strictly* convex, so that their demands are well defined. Such an assumption is in any case necessary to make the classical theory meaningful, since well-defined demands are needed to define the result of price-taking on the part of the ocean.

close[8] to 2. In Example A this leads to a bundle close to (3/4, 1/2) for the monopolist, which he prefers to his competitive bundle. In Examples B and C it leads to a bundle close to the competitive one.

The objection against this kind of approach is that it treats the ocean and the monopolist in a fundamentally asymmetric fashion. Specifically, at what point does one start to apply this theory? When the "ocean" consists of 1,000 traders, 10 traders, 2 traders, or one trader? What one would like is a theory that is applicable in any market, and when applied to a monopoly, yields the price-taking mechanism. To put the argument differently, one feels on an intuitive, common sense level that the monopolist has a distinct advantage; but economic theory, rather than explaining this phenomenon, simply states it in a specific form. For an *explanation*, one looks to game theory; but evidently, the game-theoretic notion of core is not the proper vehicle for such an explanation.

Let us try to investigate *why* the core fails to display the monopolist's power. Most of the usual objections against the core concept are not relevant here, because they argue that the core is too small, whereas in our case it is clearly too large. For example, it is sometimes argued that the definition of the core does not take coalition-forming costs into account. But such costs make it more difficult for a coalition to block, i.e., they enlarge the core; whereas we wish to find a reason to make it smaller.

The concept of core is based on what a coalition can guarantee for itself. Monopoly power is probably not based on this at all, but rather on what the monopolist can *prevent* other coalitions from getting. His strength lies in his threat possibilities, in the bargaining power engendered by the harm he can cause by refusing to trade. Put differently, the monopolist's power—and for that matter, that of any other trader—is measured by the difference between what others can get with him and what they can get without him. This line of reasoning is entirely different from that used in the definition of core. But it is not foreign to game theory; indeed, it is closely related to the ideas underlying the Shapley value. It is known that the Shapley value is significant in some economic contexts[9], and it may well turn out to be significant in accounting for monopoly power as well.

Acknowledgments

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[8] If $p^1/p^2 = 2$, the ocean's demand is not defined; see the previous footnote. As we have already noted, it is easy to modify our examples so that the preferences are strictly convex, in which case this problem will not arise.

[9] See, for example, Ref. [2].

examples. Thanks are also due to P. Champsaur, G. Laroque, and a referee for pointing out two errors in a previous version.

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