

Reply to Margalit and Yaari

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On the issue of 'logical omniscience' – the assumption that if one knows something, then one knows all its logical consequences – I am in complete agreement with Margalit and Yaari. There is no question that this is an important, far-reaching, and rather unreasonable assumption, and that it would be highly desirable to get rid of it. Unfortunately, I know of no coherent system of epistemic logic that does so, even in the one-agent case. One could take the position that this problem is so central that until it is solved, there is no point in bothering with the epistemic foundations of interactive decision theory at all. In the 'Notes on Interactive Epistemology' (Aumann, 1992) to which Margalit and Yaari refer, I chose not to take that position, but to set forth the epistemic foundations of game theory and rational economics as they appear today, i.e., with the assumption of logical omniscience.

Also, I agree entirely that the 'sensory – analytic dichotomy' is not useful or appropriate. The word 'sensory' does not appear in Aumann (1992): indeed, there is nothing there about the source of an agent's knowledge. The main point of Aumann (1992) is to give a coherent foundation for the representation of non-tautological knowledge that agents may have about each other's knowledge. If, as Margalit and Yaari claim, the dichotomy precludes agents from acquiring such knowledge, I am certainly with them in rejecting it.

On the other hand, the interpretation of 'Agreeing to Disagree' (Aumann, 1976) in Margalit and Yaari misses the whole point of that result. They write that 'Alice must take care not to condition on anything that Bob does not know'. I had hoped to have made it clear – both in the 1992 notes and in the original paper (1976) – that that is not so! The first two sentences of Aumann (1976) read, 'If two people have the same priors, and their posteriors for a given event A are common knowledge, then these two posteriors must be equal. *This is so even though they may base their posteriors on quite different information.* See also section 5 of Aumann (1992).

As for the 'Hangman's Paradox', the paradox seems to have nothing to do with *multiple* periods: it works just as well for one. Suppose one tells a class, 'Our next scheduled meeting will be devoted to an exam, but you will not know this beforehand'. Obviously, this involves a contradiction, so the students will not know what to make of it. At the next scheduled meeting, the exam is given, so the first part of the announcement was correct. The second part was also correct: since the announcement was gibberish, it couldn't be taken seriously, so the students indeed did not know about the exam.

With 17 periods, it is the same: though the announcement is more complicated to think through, in the end it is still (i) gibberish, and (ii) correct.

More precisely, it is not the content of the announcement that is gibberish, but its being an announcement – something that the listeners are meant to know after hearing. Some propositions simply cannot be known, though they may well be true – for example, any proposition of the form ' x and not kx ' (of which the one-period Hangman's Paradox is an instance). If we call this f , then kf is equivalent to ' kx and

not kx ', which is tautologically false.

While I agree with Margalit and Yaari on the 'sensory – analytic dichotomy', I must admit to being puzzled as to the nature of the point they are trying to make. If Alice tells Bob that the light turned green, why doesn't he know this? They say that he only knows that she said it: but that sounds like a quibble. If one wishes to go that route, one may as well say that she herself doesn't know that the light is green, only that she sees it as green; perhaps she is hallucinating or dreaming, or has developed a fever that affects her colour perception. Alternatively put, why do Margalit and Yaari not consider hearing as one of the senses? This whole line of argument is unconvincing. In particular, the conclusion that knowledge cannot be conveyed between individuals seems farfetched indeed.

It appears from several places in their chapter (e.g., p. 91, lines 25–31; p. 92, lines 30–32; p. 94, lines 16–18) that Margalit and Yaari have a basic misunderstanding of the construction in Aumann (1992). I *assume* no analytic knowledge other than that which is embodied in the axiom schema of the propositional calculus and of knowledge, and the *modus ponens* rule. All other analytic knowledge is *derived*, not assumed. Given the number of agents, the state space is *categoric*. And if one considers the number of agents an issue, it may be taken to be a denumerable infinity (i.e., a pool large enough to cover all agents); then this, too, is categoric, and there is no substantive knowledge at all in the state space. In a sense, that is the whole point of the construction in Aumann (1992) – to *derive* the state space, so that it need *not* be assumed as analytic. The problems that Margalit and Yaari raise are discussed in section 5 of my 1992 notes; the subsequent construction solves these problems.

One may also think of this in another way: that the state space is not a part of the agent's knowledge at all, but is only in the analyst's mind. All that the agent knows is a list of formulas. The rest is a construction that is useful for the analyst, but that the agent does not need.

Finally, it may be that the disagreement between the authors and me is largely semantic – I use the term 'knowledge' in a more comprehensive way than they, to include what they call 'comprehension'.

There is also a difference in emphasis and direction between us. They are interested in the sources and meaning of knowledge – how things get to be known, and what it 'really means' for them to be known. My approach is less philosophical and more pragmatic; basically, I wish only to provide a coherent formalization of those aspects of knowledge that are relevant to interactive decision theory and related disciplines.

References

- Aumann, R.J. (1976) 'Agreeing to Disagree', *The Annals of Statistics*, vol. 4, pp. 1236–9.
- Aumann, R.J. (1992) 'Notes on Interactive Epistemology', version of 17 July 1992; available as DP 67 at the Center for Rationality and Interactive Decision Theory, the Hebrew University of Jerusalem, February 1995.